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Corresponding Author:
Jamilu Auwalu Adamu
National Mathematical Centre, Abuja – Nigeria
Email: whitehorseconsult@yahoo.com,
Contact: +2348038679094, +2348172234745

QR Code for Mobile users

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Banking and Economic Advanced Stressed Credit Rating Models

Jamilu Auwalu Adamu
National Mathematical Centre, Abuja – Nigeria

Abstract:
It would be recalled that Basel III was issued by Basel Committee on Banking Supervision (BCBS) as a comprehensive response to the global Credit Crisis. Stressing Risk Parameters especially Probability of Default (PD) and Loss Given Default (LGD) is just like improving Basel II and III accords and by extension the entire Credit Risk Management Profession.

Also, the existing Capital Requirements and Credit Rating Models based on the available literature failed to incorporate the probability of unforeseen adverse future events, that is the low – probability, high – impact events or typical black swans.

However, Jamilu (2015) in his paper entitled: Banking and Economic Advanced Stressed Probability of Default Models, attempted to incorporate fat – tailed effects in Logit and Probit Models by stressing and incorporated ONLY the probability distribution of the underlying stock returns (Log – Logistic 3P) and U.S. fundamental macroeconomic indicators.

In this paper, the Author proposed additional fat – tailed effects prediction models reference to existing BLACK – SCHOLES, KMV – MERTON and NAÏVE KMV – MERTON Models for calculating Default Probabilities and Recovery Rates.

Finally, the New Advanced Stressed Capital Requirements and Credit Rating Models were found efficiently working and have the ability to accurately traces the typical trajectories of the past and future economic and financial crises (Black Swans) reference to the Capital Requirements and Credit Rating Models otherwise make the models more sophisticated and extraordinary than the existing once.

Keywords: Contractional - Expansional, Black – Scholes, KMV – Merton, Naïve KMV – Merton, Black Swan, Jameel.

INTRODUCTION:

Nassim N. Taleb (2011), emphases in most of his papers, the **effects of Low – Probability, High – impact Events** and **incompleteness of prediction models** to accurately capture Financial and Economic crises or chaotic situations in the other fields of knowledge. Nassim N Taleb is the one of the leading actors in the financial markets to propagate the inclusion of **"fat – tail effects"** in the modeling of credit risk prediction models. This is of course gave birth to his popular **"Black Swan Idea or Theory"**. He seriously criticizes the **assumption of Normality** in the most of the currently useful Banking and Economic models and in the same vein, the **Black Box assumptions** of other probability distributions in the modeling process of financial models.

However, In this paper, in an attempt to predict **Black Swan Events,** the Author proposed additional Advanced Stressed models reference existing popular **Black Scholes, KMV – Merton and Naïve KMV – Merton Models** for calculating Probabilities of Default and Recovery Rates, Capital Requirements and Credit Rating Models which incorporated fat – tailed effects of the underlying Research Company Stock Returns Probability Distribution.

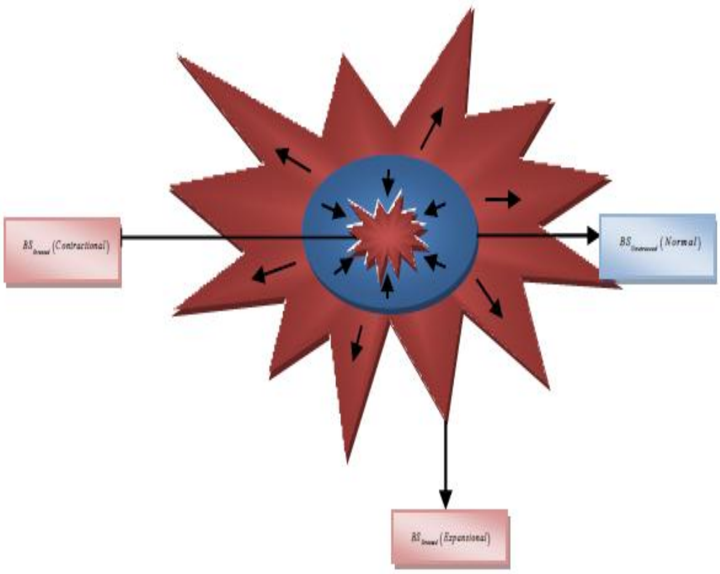
Also, the Author used the data of **Chevron Corporation** obtained from yahoo finance and Economic Research in the case of U.S. Fundamental Macroeconomic Indicators.

MATERIAL AND METHODS

The methodology adopted in this research work is to consider the traditional **Black Scholes, KMV – Merton and Naïve KMV – Merton Models** for calculating Probabilities of Default and Recovery Rates, Capital Requirements and Credit Rating Models and to develop new advanced stressed extended versions that have the ability to capture **Low - Probability, High – Impact** events popularly known as **Black Swans.**

The **IDEA** was basically on how to **Contractionally** and **Expansionally** Stress **Black – Scholes – Merton** Options Pricing Model using the respectively Geometric Volatility

σ_A and Geometric Return μ_A of the Arithmetic Means of the Underlying Asset Return and Returns of the explained (Independent) variables as well as the Best Fitted Fat – Tailed Effects Probability Distribution of the Underlying Asset Return as shown below:



The Author considers the **U.S. Fundamental Macroeconomic Indicators (Independent Variables)**, since **Chevron Corporation (CVX)** was listed in the platform of **New York Stock Exchange (NYSE)**. These include: (a) The Monthly Stock Returns of the Chevron Corporation (b) The U.S. GDP (c) The U.S. Inflation Rate (d) The U.S. Prime Rate (e) The U.S. unemployment Rate (f) The U.S. USD/GBP Exchange Rate (g) The U.S. House Price (h) The U.S. Oil Price (i) The U.S. Gold Price.

Proposed Jameel's Models VII:

The proposed models considering **Black – Scholes – Merton (1973)** Default Probability Model are given by:

TYPE A: $PD_{Stressed} = \Phi(-\mu_A J) \pm \sigma_A f(x, \mu_{Company}, \sigma_{Company}, \xi)$, **TYPE B:** $PD_{Stressed} = \Phi(-\mu_A J) \pm f(x, \mu_{Company}, \sigma_{Company}, \xi)$

TYPE C: $PD_{Stressed} = \Phi(-J) \pm \sigma_A f(x, \mu_{Company}, \sigma_{Company}, \xi)$,

TYPED: $PD_{Stressed} = \Phi(-J) \pm f(x, \mu_{Company}, \sigma_{Company}, \xi)$

Where, $PD = \Phi\left(\frac{-\left(\ln\frac{A_t}{L} + \mu_V - \frac{\sigma_V^2}{2}(T-t)\right)}{\sigma_V\sqrt{T-t}}\right)$,

$$J = \frac{\left(\ln\frac{A_t}{L} + \mu_V - \frac{\sigma_V^2}{2}(T-t)\right)}{\sigma_V\sqrt{T-t}}$$ then $PD = \Phi(-J)$.

Proposed Jameel's Models VIII:

The proposed models considering **Merton (1974)** Recovery Rate Model are given by:

TYPE A: $RR_{Stressed} = \frac{A_0}{D}\exp(\mu_V T)\frac{\Phi(-\mu_A . d_1)}{\Phi(-\mu_A . d_2)} \pm \sigma_A f(x, \mu_{Company}, \sigma_{Company}, \xi)$,

TYPE B: $RR_{Stressed} = \frac{A_0}{D}\exp(\mu_V T)\frac{\Phi(-\mu_A . d_1)}{\Phi(-\mu_A . d_2)} \pm f(x, \mu_{Company}, \sigma_{Company}, \xi)$

TYPE C: $RR_{Stressed} = \frac{A_0}{D}\exp(\mu_V . T)\frac{\Phi(-d_1)}{\Phi(-d_2)} \pm \sigma_A f(x, \mu_{Company}, \sigma_{Company}, \xi)$,

TYPE D: $RR_{Stressed} = \frac{A_0}{D}\exp(\mu_V . T)\frac{\Phi(-d_1)}{\Phi(-d_2)} \pm f(x, \mu_{Company}, \sigma_{Company}, \xi)$

Where, $RR = \frac{A_0}{D}\exp(\mu_V T)\frac{\Phi(-d_1)}{\Phi(-d_2)}$,

$$d_1 = \frac{\left(\ln\frac{A_t}{L} + \mu_V - \frac{\sigma_V^2}{2}T\right)}{\sigma_V\sqrt{T}}$$ and $d_2 = d_1 - \sigma_V\sqrt{T}$

Proposed Jameel's Models IX:

The proposed models considering **KMV – Merton** Default Probability Model are given by:

TYPE A: $PD_{Stressed} = \Phi(-\mu_A . DD) \pm \sigma_A f(x, \mu_{Company}, \sigma_{Company}, \xi)$,

TYPE B: $PD_{Stressed} = \Phi(-\mu_A . DD) \pm f(x, \mu_{Company}, \sigma_{Company}, \xi)$

TYPE C: $PD_{Stressed} = \Phi(-DD) \pm \sigma_A f(x, \mu_{Company}, \sigma_{Company}, \xi)$,

TYPE D: $PD_{Stressed} = \Phi(-DD) \pm f(x, \mu_{Company}, \sigma_{Company}, \xi)$

Where, the Distance to Default can be calculated as:

$$DD = \frac{\ln\left(\frac{V}{F}\right) + (\mu - 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}}$$

Where μ is an estimate of the expected annual return of firm's assets. The corresponding implied Probability of Default, sometimes called Expected Default Frequency (or

EDF) is given by:

$$\pi KMV = \Phi\left(-\frac{\ln\left(\frac{V}{F}\right)+\left(\mu-0.5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)$$

and $\pi KMV = \Phi\left(-DD\right)$

Existing Model (Naïve KMV – Merton Alternative)

Proposed Jameel's Models X:

The proposed models considering **Naïve KMV – Merton** Default Probability Model are given by:

TYPE A: $PD_{Stressed} = \Phi\left(-\mu_A . Naive\,DD\right) \pm \sigma_A f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$,

TYPE B: $PD_{Stressed} = \Phi\left(-\mu_A . Naive\,DD\right) \pm f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$

TYPE C: $PD_{Stressed} = \Phi\left(-Naive\,DD\right) \pm \sigma_A f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$,

TYPE D: $PD_{Stressed} = \Phi\left(-Naive\,DD\right) \pm f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right)$

Where, the Distance to Default can be calculated as:

$$DD = \frac{\ln\left(\frac{V}{F}\right)+\left(\mu-0.5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}$$

Where μ is an estimate of the expected annual return of firm's assets.The corresponding implied Probability of Default, sometimes called Expected Default Frequency (or EDF) is given by:

$$\pi KMV = \Phi\left(-\frac{\ln\left(\frac{V}{F}\right)+\left(\mu-0.5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)$$ and

$\pi KMV = \Phi\left(-DD\right)$ with the substitutions that the volatility of each firm' debt is given by: $Naive\ \sigma_D = 0.05 + 0.25\sigma_E$ and $Naive\ \mu = r_{u-1}$

By allowing Naïve estimate of μ to depends on past returns, we incorporate the same information. The Naïve Distance to Default is given by:

$$Naive\ DD = \frac{\ln\left[\frac{(E+F)}{F}\right] + \left(r_{u-1} - 0.5\,Naive\,\sigma_V^2\right)T}{Naive\,\sigma_V\sqrt{T}}$$

.Naïve Probability estimate is given by:

$\pi_{Naive} = \Phi\left(-Naive\ DD\right)$

PROPOSED MODELS ON STRESSED MIGRATION MATRICES

Under Credit Risk, an accurate computation of probabilities entries of Transition Matrices is very necessary, since they contain vital information about the migration probabilities of the debtors moving from one credit class to another or probability to stay in the actual creditworthiness class within a certain observation period.

M. Beitz and M. Ehrhardt (2010) stated that "In average transition matrices, the probability to migrate in better creditworthiness classes are too pessimistically in times of economic recoveries, while these probabilities are overestimated in times of economic recessions".

Forest, Belkin and Suchower (1998), developed One-Parameter Representation method for calculating average transition matrices, however, Beitz and M. Ehrhardt (2010) criticized and argue that the methodology lack hope at the times of economic recoveries and overestimates probabilities at the times of economic recessions.

Beitz and M. Ehrhardt (2010) proposed a new modified One-Parameter Representation version of Forest, Belkin and Suchower (1998) that is more stable and sensitive to both economic recoveries and recessions. They claimed their One-Parameter Representation possessed the following qualities to quote them precisely:

- **Lower expenditure for data mining:** Instead of a whole history of transition matrices, only an average and a single year transition matrix is needed to estimate the parameter ρ .
- **Less computing time:** The computing time decreases considerably because, only one least square estimation is needed to estimate the parameter μ_t and σ_t .
- **Better approximation:** An additional degree of freedom decreases the sum of the errors. Hence, a better approximation of the single year transition matrix is possible.

Thought both the Beitz and M. Ehrhardt (2010) and Forest, Belkin and Suchower (1998) One-Parameter Representation methods advantages did not categorically mention **FAT –TAILED EFFECTS** incorporation into the good qualities of their methodologies, nevertheless, something tangible has been done and contributed to the existing knowledge (thanks to their One-Parameter Representations).

The only problem I can see vividly from both Beitz and M. Ehrhardt (2010) and Forest, Belkin and Suchower (1998) One-Parameter Representation methods is the **FAILURE** to incorporate **FAT –TAILED EFFECTS,** hence the **NORMALITY** assumptions.

However, these methods can be **IMPROVED** by incorporating components that takes care of **FAT – TAILED EFFECTS** in them so as to categorically capture **BLACK SWAN EVENTS** (the recoveries and recessions periods or the low – probability, high – impact events). This can be done using **Jameel's Contractional and Expansional Stress Methods** by considering the existing methods one by one as follows:

Existing Models (Forest, Belkin and Suchower One-Parameter Representation Method):

Forest, Belkin and Suchower (1998) describe yield as a sum of a systematic and for a debtor's specific component:

$$X = \sqrt{1-\rho^2 Y} + \sqrt{\rho} Z$$

Y describes the debtors' specific component and Z describes the systematic, the economic influenced, part of the yield of a debtor and ρ denotes a weighting factor of both components and a measure for the correlation between the yield X and the systematic component Z. While the specific component of a debtor varies within one year, the economic component Z is a, within one year, stable quantity.

$Z(t)$ describes the value of the systematic component of the year t. During an economic recovery $Z(t)$ takes positive values. Contrary, negative Z-values results from times of economic recessions. As standard normal random variable, Z has the mathematical expectation value zero. Thus the economic situation has in average no influence on the yields of the debtors.

Following the CreditMetricsTM approach, the density of the standard normal deviation is, for every initial rating, partitioned into transition thresholds $x_{i,j}$. These thresholds were estimated by means of the probability of the transition matrices as follows:

$$x_{i,default} = \Phi^{-1}(p_{i,default})$$
$$x_{i,CCC} = \Phi^{-1}(p_{i,default} + p_{i,CCC})$$
$$x_{i,CC} = \Phi^{-1}(p_{i,CCC} + p_{i,CC})$$
$$\vdots$$

Where, Φ^{-1} is the standard normal deviation. Z(t) must be chosen such that the cyclical one-year-old migration probabilities p(t) are well approximated by:

$$\Delta\left(x_{i,j}, Z(t)\right) = \Phi\left(\frac{x_{i,j+1} - \sqrt{\rho} Z(t)}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{x_{i,j} - \sqrt{\rho} Z(t)}{\sqrt{1-\rho}}\right)$$

They solve the above problem by using a modified least square method and obtained the following transition probability $P(i, j, Z(t))$ of the year t is, for every initial rating i and final rating j, given by:

$$P\left(i, j, Z(t)\right) = \Phi\left(\frac{x_{i,j+1} - \sqrt{\rho} Z(t)}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{x_{i,j} - \sqrt{\rho} Z(t)}{\sqrt{1-\rho}}\right)$$

Proposed Jameel's Models XI

Recall that following the CreditMetricsTM approach, the density of the standard normal deviation is, for every initial rating, partitioned into transition thresholds $x_{i,j}$. These thresholds were estimated by means of the probability of the transition matrices as follows:

$$x_{i,default} = \Phi^{-1}(p_{i,default})$$
$$x_{i,CCC} = \Phi^{-1}(p_{i,default} + p_{i,CCC})$$
$$x_{i,CC} = \Phi^{-1}(p_{i,CCC} + p_{i,CC})$$
$$\vdots$$

Let μ_A and σ_A be respectively the Geometric Mean and Volatility of Arithmetic Means and Volatilities of the Research Country's Fundamental Macroeconomic Indicators plus the Stock Return of the Company under consideration.

Let $f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$ be a Log – Logistic (3P) or any other **BEST FITTED FAT – TAILED EFFECTS** probability distribution of the Research Company Stock Return then applying **Jameel's Contractional and Expansional Stress Methods** on these thresholds, we obtained the following:

Forest, Belkin and Suchower Method :

$$x_{i,default} = \Phi^{-1}\left(p_{i,default}\right)$$
$$p_{i,default} = \Phi\left(x_{i,default}\right)$$

Equivalent Jameel's Methods :

$$p_{i,default(Stressed)} = \Phi\left(\mu_A \times x_{i,default}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$
$$p_{i,default(Stressed)} = \Phi\left(\mu_A \times x_{i,default}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$
$$p_{i,default(Stressed)} = \Phi\left(x_{i,default}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$
$$p_{i,default(Stressed)} = \Phi\left(x_{i,default}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Forest, Belkin and Method :

$$x_{i,CCC} = \Phi^{-1}\left(p_{i,default} + p_{i,CCC}\right)$$
$$p_{i,default} + p_{i,CCC} = \Phi\left(x_{i,CCC}\right)$$

Equivalent Jameel's Methods :

$$p_{i,default(Stressed)} + p_{i,CCC(Stressed)} = \Phi\left(\mu_A \times x_{i,CCC}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$
$$p_{i,default(Stressed)} + p_{i,CCC(Stressed)} = \Phi\left(\mu_A \times x_{i,CCC}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$
$$p_{i,default(Stressed)} + p_{i,CCC(Stressed)} = \Phi\left(x_{i,CCC}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$
$$p_{i,default(Stressed)} + p_{i,CCC(Stressed)} = \Phi\left(x_{i,CCC}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

$$\Rightarrow \begin{cases} p_{i,CCC(Stressed)} = \Phi\left(\mu_A \times x_{i,CCC}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right) - p_{i,default(Stressed)} \\ p_{i,CCC(Stressed)} = \Phi\left(\mu_A \times x_{i,CCC}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right) - p_{i,default(Stressed)} \\ p_{i,CCC(Stressed)} = \Phi\left(x_{i,CCC}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right) - p_{i,default(Stressed)} \\ p_{i,CCC(Stressed)} = \Phi\left(x_{i,CCC}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right) - p_{i,default(Stressed)} \end{cases}$$

Forest, Belkin and Suchower Method :

$$x_{i,CC} = \Phi^{-1}\left(p_{i,CCC} + p_{i,CC}\right)$$

$$p_{i,default} + p_{i,CCC} = \Phi\left(x_{i,CCC}\right)$$

Equivalent Jameel's Methods:

$$p_{i,CCC(Stressed)} + p_{i,CC(Stressed)} = \Phi\left(\mu_A \times x_{i,CC}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

$$p_{i,CCC(Stressed)} + p_{i,CC(Stressed)} = \Phi\left(\mu_A \times x_{i,CC}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

$$p_{i,CCC(Stressed)} + p_{i,CC(Stressed)} = \Phi\left(x_{i,CC}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

$$p_{i,CCC(Stressed)} + p_{i,CC(Stressed)} = \Phi\left(x_{i,CC}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

$$\Rightarrow \begin{Bmatrix} p_{i,CC(Stressed)} = \Phi\left(\mu_A \times x_{i,CC}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right) - p_{i,CCC(Stressed)} \\ p_{i,CC(Stressed)} = \Phi\left(\mu_A \times x_{i,CC}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right) - p_{i,CCC(Stressed)} \\ p_{i,CC(Stressed)} = \Phi\left(x_{i,CC}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right) - p_{i,CCC(Stressed)} \\ p_{i,CC(Stressed)} = \Phi\left(x_{i,CC}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right) - p_{i,CCC(Stressed)} \end{Bmatrix}$$

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Such that the transition probability $P\left(i, j, Z(t)\right)$ of the year t is, for every initial rating i and final rating j, given by:

Forest, Belkin and Suchower Method :

$$P\left(i, j, Z(t)\right) = \Phi\left(\frac{x_{i,j+1} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{x_{i,j} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right)$$

Equivalent Jameel's Methods :

TYPE A:

$$P\left(i, j, Z(t)\right)_{Stressed} = \Phi\left(\mu_A\left[\frac{x_{i,j+1} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) - \Phi\left(\mu_A\left[\frac{x_{i,j} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE B*:

$$P\left(i, j, Z(t)\right)_{Stressed} = \Phi\left(\mu_A\left[\frac{x_{i,j+1} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) - \Phi\left(\left[\frac{x_{i,j} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE C:

$$P\left(i, j, Z(t)\right)_{Stressed} = \Phi\left(\mu_A\left[\frac{x_{i,j+1} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) - \Phi\left(\mu_A\left[\frac{x_{i,j} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE D*:

$$P\left(i, j, Z(t)\right)_{Stressed} = \Phi\left(\mu_A\left[\frac{x_{i,j+1} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) - \Phi\left(\left[\frac{x_{i,j} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE E:

$$P\left(i, j, Z(t)\right)_{Stressed} = \Phi\left(\left[\frac{x_{i,j+1} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) - \Phi\left(\left[\frac{x_{i,j} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE F:

$$P\left(i, j, Z(t)\right)_{Stressed} = \Phi\left(\left[\frac{x_{i,j+1} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) - \Phi\left(\left[\frac{x_{i,j} - \sqrt{\rho}Z(t)}{\sqrt{1-\rho}}\right]\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Existing Models (M. Beitz and M. Ehrhardt One-Parameter Representation Method)

For the sake of convenience, M. Beitz and M. Ehrhardt (2010), regard the case, that the stock yield X_t of the special year t is normal distributed with mathematical expectation value μ_t and variance σ_t^2.

To determine the parameters μ_t and σ_t of the distribution of the stock yields in a certain year t, it is necessary to estimate the threshold values $x_{i,j}$ of the average migration probabilities. They approximate the one-year-old transition probability by means of the average transition probability, adjusted by μ_t and σ_t^2 as follows:

$$\Delta\left(x_{i,j}, \mu_t, \sigma_t^2\right) = \Phi\left(\frac{x_{i,j+1} - \mu_t}{\sigma_t}\right) - \Phi\left(\frac{x_{i,j} - \mu_t}{\sigma_t}\right)$$

The migration probability $P\left(i, j, \mu_t, \sigma_t^2\right)$ of a year which is influenced by the economic situation can be derived for every combination of initial rating i and final rating j as follows:

$$P\left(i, j, \mu_t, \sigma_t^2\right) = \Phi\left(\frac{x_{i,j+1} - \mu_t}{\sigma_t}\right) - \Phi\left(\frac{x_{i,j} - \mu_t}{\sigma_t}\right)$$

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TYPE A:

$$P\left(i, j, \mu_t, \sigma_t^2\right)_{Stressed} = \Phi\left(\mu_A\left[\frac{x_{i,j+1} - \mu_t}{\sigma_t}\right]\right) - \Phi\left(\mu_A\left[\frac{x_{i,j} - \mu_t}{\sigma_t}\right]\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE B*:

$$P\left(i, j, \mu_t, \sigma_t^2\right)_{Stressed} = \Phi\left(\mu_A\left[\frac{x_{i,j+1} - \mu_t}{\sigma_t}\right]\right) - \Phi\left(\left[\frac{x_{i,j} - \mu_t}{\sigma_t}\right]\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE C:

$$P\left(i, j, \mu_t, \sigma_t^2\right)_{Stressed} = \Phi\left(\mu_A\left[\frac{x_{i,j+1} - \mu_t}{\sigma_t}\right]\right) - \Phi\left(\mu_A\left[\frac{x_{i,j} - \mu_t}{\sigma_t}\right]\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE D*:

$$P\left(i, j, \mu_t, \sigma_t^2\right)_{Stressed} = \Phi\left(\mu_A\left[\frac{x_{i,j+1} - \mu_t}{\sigma_t}\right]\right) - \Phi\left(\left[\frac{x_{i,j} - \mu_t}{\sigma_t}\right]\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE E:

$$P\left(i, j, \mu_t, \sigma_t^2\right)_{Stressed} = \Phi\left(\left[\frac{x_{i,j+1} - \mu_t}{\sigma_t}\right]\right) - \Phi\left(\left[\frac{x_{i,j} - \mu_t}{\sigma_t}\right]\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE F:

$$P\left(i, j, \mu_t, \sigma_t^2\right)_{Stressed} = \Phi\left(\left[\frac{x_{i,j+1} - \mu_t}{\sigma_t}\right]\right) - \Phi\left(\left[\frac{x_{i,j} - \mu_t}{\sigma_t}\right]\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Existing Model (Robert J. Powell and David E. Allen Model):

Robert J. Powell and David E. Allen (2009) define conditional probability of default (CPD) as being PD on the condition that standard deviation of asset returns exceeds standard deviation at the 95% confidence level, i.e. the worst 5% of asset returns. They calculate the standard deviation of the worst 5% of daily asset returns for each period to obtain a conditional standard deviation (CStdev).

We then substitute CStdev into the formula used to calculate DD, to obtain a conditional DD (CDD). CPD is calculated by substituting DD with CDD into the CPD formula.

$$CDD = \frac{\ln\left(\frac{V}{F}\right) + \left(\mu - 0.5\sigma_V^2\right)T}{CStdev_V \sqrt{T}}$$ and

$$CPD = \Phi\left(-CDD\right)$$

Jameel's Observations:

- The value of α has being chosen arbitrarily. What if the value of α is slightly **HIGHER** or **LOWER** than what we have chosen?
- The CPD is assumed to be **NORMALLY** distributed.
- No any **FAT – TAILED EFFECTS** (components) are incorporated in the model

Proposed Jameel's Models XIII (reference Robert J. Powell and David E. Allen Conditional Probability of Default):

TYPE A:

$$CPD_{Stressed} = \Phi\left(-\mu_A CDD\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE B:

$$CPD_{Stressed} = \Phi\left(-\mu_A CDD\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE C:

$$CPD_{Stressed} = \Phi\left(-CDD\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE D:

$$CPD_{Stressed} = \Phi\left(-CDD\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Existing Model (reference to Basel II Asymptotic Single Risk Factor Model):

Based on the Basel II Asymptotic Single Risk Factor (ASRF) Model, we have the following relations in respect to Default Probabilities:

$$PD_i \setminus \left\{Z_t = \Phi^{-1}(\alpha)\right\} = \Phi\left[\frac{\Phi^{-1}\left(\overline{p_i}\right) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right]$$

Where $PD_i \setminus \{...\}$ is the conditional default probability in state Z, Φ is the Gaussian cumulative distribution function, $\overline{p_i}$ is the long term average probability of default (PD) of class i $(i = 1, ..., n)$ (or unconditional PD) and α is the probability that the value Z (or below). The closer α is to 0, the less frequent is the crisis and the greater its severity: in the Basel II framework, α **is equal to 0.1%** (this corresponds to the regulatory confidence level of 99.9%) which is the required confidence level to compute regulatory capital requirement under **Basel II and III frameworks**.

To compute the probability of transition from rating class i to rating class j, the above formula, expanded to every component of the $n \times n$ $(here\ 8 \times 8)$, the transition Matrix yield:

$$P_{ijt} = \Phi\left[\frac{\Phi^{-1}\left(\overline{p_{i8}} + ... + \overline{p_{ij}}\right) + \sqrt{\rho}\,Z_t}{\sqrt{1-\rho}}\right] - P_{i8t} - ... - P_{i,j+1,t}$$

Jameel's Observations:

- Considering the probability of default obtained using Asymptotic Single Risk Factor (ASRF) Model above, it seems the value of α is chosen arbitrarily and even under Basel II framework α was chosen to be 0.1% which I think there is no definite methodology (model) used to arrived at such value, however, what if we choose α to be 0.1% and the crisis severity is more than 0.1%? still we cannot discard the choice of α (Thanks to Basel II), however, the model need to be improved
- The conditional default probability $PD_i \setminus \{...\}$ assumed **NORMALITY**
- No any **FAT – TAILED EFFECTS** (components) are incorporated in the model

Hence, Jameel proposed the following models in respect to conditional default probability and probability of transition from rating class i to rating class j:

Proposed Jameel's Models 14 (reference to ASRF Conditional Default Probability):

TYPE A:

$$PD_i \setminus \left\{Z_t = \Phi^{-1}(\alpha)\right\}_{Stressed} = \Phi\left[\mu_A\left(\frac{\Phi^{-1}\left(\overline{p_i}\right) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right)\right] \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE B:

$$PD_i \setminus \left\{Z_t = \Phi^{-1}(\alpha)\right\}_{Stressed} = \Phi\left[\mu_A\left(\frac{\Phi^{-1}\left(\overline{p_i}\right) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right)\right] \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE C:

$$PD_i \setminus \left\{Z_t = \Phi^{-1}(\alpha)\right\}_{Stressed} = \Phi\left(\frac{\Phi^{-1}\left(\overline{p_i}\right) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE D:

$$PD_i \setminus \left\{Z_t = \Phi^{-1}(\alpha)\right\}_{Stressed} = \Phi\left(\frac{\Phi^{-1}\left(\overline{p_i}\right) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Proposed Jameel's Models 15 (Probability of Transition from Rating Class i to Rating Class j):

TYPE A:

$$P_{ijt(Stressed)} = \Phi\left[\mu_A\left(\frac{\Phi^{-1}\left(\overline{p_{i8}} + ... + \overline{p_{ij}}\right) + \sqrt{\rho}\,Z_t}{\sqrt{1-\rho}}\right)\right] - P_{i8t} - ... - P_{i,j+1,t} \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE B:

$$P_{ijt(Stressed)} = \Phi\left[\mu_A\left(\frac{\Phi^{-1}\left(\overline{p_{i8}} + \ldots + \overline{p_{ij}}\right) + \sqrt{\rho} Z_t}{\sqrt{1-\rho}}\right)\right] - P_{i8t} - \ldots - P_{i,j+1,t} \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE C:

$$P_{ijt(Stressed)} = \Phi\left(\frac{\Phi^{-1}\left(\overline{p_{i8}} + \ldots + \overline{p_{ij}}\right) + \sqrt{\rho} Z_t}{\sqrt{1-\rho}}\right) - P_{i8t} - \ldots - P_{i,j+1,t} \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE D:

$$P_{ijt(Stressed)} = \Phi\left(\frac{\Phi^{-1}\left(\overline{p_{i8}} + \ldots + \overline{p_{ij}}\right) + \sqrt{\rho} Z_t}{\sqrt{1-\rho}}\right) - P_{i8t} - \ldots - P_{i,j+1,t} \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Existing Model (CreditMetrics and CreditRisk + Conditional Default Probability)

The CreditMetrics Conditional Default Probability, given a realization of the factor Z, that a counterparty's asset return falls below α is given by:

$$P(Z) = \Phi\left(\frac{\alpha - \sqrt{\rho} Z}{1-\rho}\right)$$

The CreditRisk + Conditional Default Probability, given a realization of the factor Z, that a counterparty's asset return falls below α is given by:

$$P(Z) = \Phi\left(\frac{\alpha - \sqrt{\sigma} Z}{1-\sigma}\right)$$

Where Φ is the normal cumulative probability distribution and $\sigma = Std\ Dev\left[P(Z)\right]$.

Jameel's Observations:

- The value of α has being chosen arbitrarily. What if the value of α is slightly **HIGHER** or **LOWER** than what we have chosen?
- The conditional default probability $P(Z)$ assumed **NORMALITY**
- No any **FAT – TAILED EFFECTS** (components) are incorporated in the model

Proposed Jameel's Models 16 (reference to CreditMetrics):

TYPE A:

$$P(Z)_{Stressed} = \Phi\left[\mu_A\left(\frac{\alpha - \sqrt{\rho} Z}{1-\rho}\right)\right] \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE B:

$$P(Z)_{Stressed} = \Phi\left[\mu_A\left(\frac{\alpha - \sqrt{\rho} Z}{1-\rho}\right)\right] \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE C:

$$P(Z)_{Stressed} = \Phi\left(\frac{\alpha - \sqrt{\rho} Z}{1-\rho}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE D:

$$P(Z)_{Stressed} = \Phi\left(\frac{\alpha - \sqrt{\rho} Z}{1-\rho}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Proposed Jameel's Models 17 (reference to CreditRisk +):

TYPE A:

$$P(Z)_{Stressed} = \Phi\left[\mu_A\left(\frac{\alpha - \sqrt{\sigma} Z}{1-\sigma}\right)\right] \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE B:

$$P(Z)_{Stressed} = \Phi\left[\mu_A\left(\frac{\alpha - \sqrt{\sigma} Z}{1-\sigma}\right)\right] \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE C:

$$P(Z)_{Stressed} = \Phi\left(\frac{\alpha - \sqrt{\sigma} Z}{1-\sigma}\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE D:

$$P(Z)_{Stressed} = \Phi\left(\frac{\alpha - \sqrt{\sigma} Z}{1-\sigma}\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Existing Models (Through – the – Cycle and Point – in - Time PDs):

Recall that for calculation of Capital buffer against Unexpected losses, the Through – the – Cycle (TTC) PD (unconditional of the states of economic cycle, PD) should be used in the RWA formulas.

We can calculate an Obligor's TTC PD in the case of 100% PIT Rating model as follows:

$$TTC - PD_i = \Phi\left[\alpha\sqrt{\rho} Z_{now} + \sqrt{1-\alpha^2\rho}\ \Phi^{-1}\left(PD_{i,\alpha}\right)\right]$$

Assuming $\alpha\ \%\ \ (not\ 100\%)$ PTT PD rating model which means that part of economy effect is already averaged in rating model PD. For the limiting cases alpha = 0 or alpha = 1, we have TTC and PIT PD model similar to 100% PIT PD model.

$$PIT - PD_i(Z) = \Phi\left[\frac{\left(B_i - \sqrt{\rho} Z\right)}{\sqrt{1-\rho}}\right]$$

Where, $B_i = \sqrt{\rho}\ \alpha\ Z_{now} + \sqrt{1-\rho\,\alpha^2}\ \Phi^{-1}\left(PD_{i,\alpha}\right)$.

If $i = 1$ from PIT formula, we obtained what we called **Default Rate in global portfolio** given by:

$$DF = \Phi\left[\frac{\left(B - \sqrt{\rho} Z\right)}{\sqrt{1-\rho}}\right]$$

Where B can be interpreted as belonging to average or "central tendency" client of the global portfolio. Economy state Z and correlation ρ can be calculated by using maximum likelihood or method of moments.

Also, we can use CreditMetrics Single – Factor model to determine PIT EDFs as follows:

$$PD_{gt} = \Phi\left(\frac{F^{-1}(PD_g) - \sqrt{\rho}Z_t}{\sqrt{1-\rho}}\right)$$

Where, PD_{gt} is the estimated PD for grade g in period t, ρ : correlation coefficient, F : standard normal cdf, Z_t : average factor value for period t, PD_g : long - run average PD for grade g. Here, we use PDs rather than EDFs to denote one – year default rates.

Jameel's Observations:

- The value of α has being chosen arbitrarily. What if the value of α is slightly **HIGHER** or **LOWER** than what we have chosen?
- $TTC - PD_i s$ and $PIT - PD_i(Z)s$ are assumed to be **NORMALLY** distributed.
- No any **FAT – TAILED EFFECTS** (components) are incorporated in the model

Proposed Jameel's Models 18 (reference to Through – the – Cycle):

TYPE A:

$$TTC - PD_{i(Stressed)} = \Phi\left[\mu_A\left(\alpha\sqrt{\rho}Z_{now} + \sqrt{1-\alpha^2\rho}\,\Phi^{-1}(PD_{i,\alpha})\right)\right] \pm \sigma_A f(x, \mu_{company}, \sigma_{company}, \xi)$$

TYPE B:

$$TTC - PD_{i(Stressed)} = \Phi\left[\mu_A\left(\alpha\sqrt{\rho}Z_{now} + \sqrt{1-\alpha^2\rho}\,\Phi^{-1}(PD_{i,\alpha})\right)\right] \pm f(x, \mu_{company}, \sigma_{company}, \xi)$$

TYPE C:

$$TTC - PD_{i(Stressed)} = \Phi\left(\alpha\sqrt{\rho}Z_{now} + \sqrt{1-\alpha^2\rho}\,\Phi^{-1}(PD_{i,\alpha})\right) \pm \sigma_A f(x, \mu_{company}, \sigma_{company}, \xi)$$

TYPE D:

$$TTC - PD_{i(Stressed)} = \Phi\left(\alpha\sqrt{\rho}Z_{now} + \sqrt{1-\alpha^2\rho}\,\Phi^{-1}(PD_{i,\alpha})\right) \pm f(x, \mu_{company}, \sigma_{company}, \xi)$$

Proposed Jameel's Models 19 (reference to Point – in - Time):

TYPE A:

$$PIT - PD_i(Z)_{Stressed} = \Phi\left[\mu_A\left(\frac{(B_i - \sqrt{\rho}Z)}{\sqrt{1-\rho}}\right)\right] \pm \sigma_A f(x, \mu_{company}, \sigma_{company}, \xi)$$

TYPE B:

$$PIT - PD_i(Z)_{Stressed} = \Phi\left[\mu_A\left(\frac{(B_i - \sqrt{\rho}Z)}{\sqrt{1-\rho}}\right)\right] \pm f(x, \mu_{company}, \sigma_{company}, \xi)$$

TYPE C:

$$PIT - PD_i(Z)_{Stressed} = \Phi\left(\frac{(B_i - \sqrt{\rho}Z)}{\sqrt{1-\rho}}\right) \pm \sigma_A f(x, \mu_{company}, \sigma_{company}, \xi)$$

TYPE D:

$$PIT - PD_i(Z)_{Stressed} = \Phi\left(\frac{(B_i - \sqrt{\rho}Z)}{\sqrt{1-\rho}}\right) \pm f(x, \mu_{company}, \sigma_{company}, \xi)$$

Recall that PIT reflect current conditions of economic cycle, TTC insensitive for cyclical moments, while sensitive for changes in risk profile. However, with **Jameel's advanced models both TTC and PIT are improved and more sensitive to economic and financial crises**.

TTC and PIT can be used to calculate Expected Losses, Risk Weighted Assets and RAROC.

Proposed Jameel's Models 20 (reference to CreditMetrics Single – Factor model):

TYPE A:

$$PD_{gt(Stressed)} = \Phi\left[\mu_A\left(\frac{F^{-1}(PD_g) - \sqrt{\rho}Z_t}{\sqrt{1-\rho}}\right)\right] \pm \sigma_A f(x, \mu_{company}, \sigma_{company}, \xi)$$

TYPE B:

$$PD_{gt(Stressed)} = \Phi\left[\mu_A\left(\frac{F^{-1}(PD_g) - \sqrt{\rho}Z_t}{\sqrt{1-\rho}}\right)\right] \pm f(x, \mu_{company}, \sigma_{company}, \xi)$$

TYPE C:

$$PD_{gt(Stressed)} = \Phi\left(\frac{F^{-1}(PD_g) - \sqrt{\rho}Z_t}{\sqrt{1-\rho}}\right) \pm \sigma_A f(x, \mu_{company}, \sigma_{company}, \xi)$$

TYPE D:

$$PD_{gt(Stressed)} = \Phi\left(\frac{F^{-1}(PD_g) - \sqrt{\rho}Z_t}{\sqrt{1-\rho}}\right) \pm f(x, \mu_{company}, \sigma_{company}, \xi)$$

Existing Model (CreditMetrics™ (1997)):

Gupton, Finger and Bhatia (1997) of CreditMetrics™ now CreditMetrics use transition probabilities of upgrades, downgrades and defaults (referred as Credit Rating changes) and a **NORMAL** distribution assumption to calculate asset thresholds (Z) for each credit class as follows:

Rating	Probability according to the Asset Value Model
AAA	$1-\Phi\left(\frac{Z_{AA}}{\sigma}\right)$
AA	$\Phi\left(\frac{Z_{AA}}{\sigma}\right)-\Phi\left(\frac{Z_{A}}{\sigma}\right)$
A	$\Phi\left(\frac{Z_{A}}{\sigma}\right)-\Phi\left(\frac{Z_{BBB}}{\sigma}\right)$
BBB	$\Phi\left(\frac{Z_{BBB}}{\sigma}\right)-\Phi\left(\frac{Z_{BB}}{\sigma}\right)$
BB	$\Phi\left(\frac{Z_{BB}}{\sigma}\right)-\Phi\left(\frac{Z_{B}}{\sigma}\right)$
B	$\Phi\left(\frac{Z_{B}}{\sigma}\right)-\Phi\left(\frac{Z_{CCC}}{\sigma}\right)$
CCC	$\Phi\left(\frac{Z_{CCC}}{\sigma}\right)-\Phi\left(\frac{Z_{Def}}{\sigma}\right)$
Default	$\Phi\left(\frac{Z_{Def}}{\sigma}\right)$

Proposed Jameel's Models 21 (reference to CreditMetrics™ (1997)):

Types	Rating	Probability according to the Asset Value Model
	AAA	
TYPE:A		$1-\left[\Phi\left(\frac{\mu_A Z_{AA}}{\sigma}\right)\pm\sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)\right]$
TYPE:B		$1-\left[\Phi\left(\frac{\mu_A Z_{AA}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)\right]$
TYPE:C		$1-\left[\Phi\left(\frac{Z_{AA}}{\sigma}\right)\pm\sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)\right]$
TYPE: D		$1-\left[\Phi\left(\frac{Z_{AA}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)\right]$
	AA	
TYPE:A		$\Phi\left(\frac{\mu_A Z_{AA}}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_{A}}{\sigma}\right)\pm\sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:B		$\Phi\left(\frac{\mu_A Z_{AA}}{\sigma}\right)-\Phi\left(\frac{Z_{A}}{\sigma}\right)\pm\sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:C		$\Phi\left(\frac{\mu_A Z_{AA}}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_{A}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:D		

TYPE:E	$\Phi\left(\frac{\mu_A Z_{AA}}{\sigma}\right)-\Phi\left(\frac{Z_A}{\sigma}\right) \pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:F	$\Phi\left(\frac{Z_{AA}}{\sigma}\right)-\Phi\left(\frac{Z_A}{\sigma}\right) \pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
	$\Phi\left(\frac{Z_{AA}}{\sigma}\right)-\Phi\left(\frac{Z_A}{\sigma}\right) \pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
A	
TYPE:A	$\Phi\left(\frac{\mu_A Z_A}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_{BBB}}{\sigma}\right) \pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:B	$\Phi\left(\frac{\mu_A Z_A}{\sigma}\right)-\Phi\left(\frac{Z_{BBB}}{\sigma}\right) \pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:C	$\Phi\left(\frac{\mu_A Z_A}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_{BBB}}{\sigma}\right) \pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:D	$\Phi\left(\frac{\mu_A Z_A}{\sigma}\right)-\Phi\left(\frac{Z_{BBB}}{\sigma}\right) \pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:E	$\Phi\left(\frac{Z_A}{\sigma}\right)-\Phi\left(\frac{Z_{BBB}}{\sigma}\right) \pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:F	$\Phi\left(\frac{Z_A}{\sigma}\right)-\Phi\left(\frac{Z_{BBB}}{\sigma}\right) \pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
BBB	
TYPE:A	$\Phi\left(\frac{\mu_A Z_{BBB}}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_{BB}}{\sigma}\right) \pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:B	$\Phi\left(\frac{\mu_A Z_{BBB}}{\sigma}\right)-\Phi\left(\frac{Z_{BB}}{\sigma}\right) \pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:C	$\Phi\left(\frac{\mu_A Z_{BBB}}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_{BB}}{\sigma}\right) \pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:D	$\Phi\left(\frac{\mu_A Z_{BBB}}{\sigma}\right)-\Phi\left(\frac{Z_{BB}}{\sigma}\right) \pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:E	$\Phi\left(\frac{Z_{BBB}}{\sigma}\right)-\Phi\left(\frac{Z_{BB}}{\sigma}\right) \pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:F	$\Phi\left(\frac{Z_{BBB}}{\sigma}\right)-\Phi\left(\frac{Z_{BB}}{\sigma}\right) \pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
BB	
TYPE:A	$\Phi\left(\frac{\mu_A Z_{BB}}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_B}{\sigma}\right) \pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:B	$\Phi\left(\frac{\mu_A Z_{BB}}{\sigma}\right)-\Phi\left(\frac{Z_B}{\sigma}\right) \pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:C	$\Phi\left(\frac{\mu_A Z_{BB}}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_B}{\sigma}\right) \pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:D	$\Phi\left(\frac{\mu_A Z_{BB}}{\sigma}\right)-\Phi\left(\frac{Z_B}{\sigma}\right) \pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:E	$\Phi\left(\frac{Z_{BB}}{\sigma}\right)-\Phi\left(\frac{Z_B}{\sigma}\right) \pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$
TYPE:F	

	$\Phi\left(\frac{Z_{BB}}{\sigma}\right)-\Phi\left(\frac{Z_{B}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
B **TYPE:A** **TYPE:B** **TYPE:C** **TYPE:D** **TYPE:E** **TYPE:F**	$\Phi\left(\frac{\mu_A Z_{B}}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_{CCC}}{\sigma}\right)\pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{\mu_A Z_{B}}{\sigma}\right)-\Phi\left(\frac{Z_{CCC}}{\sigma}\right)\pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{\mu_A Z_{B}}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_{CCC}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{\mu_A Z_{B}}{\sigma}\right)-\Phi\left(\frac{Z_{CCC}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{Z_{B}}{\sigma}\right)-\Phi\left(\frac{Z_{CCC}}{\sigma}\right)\pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{Z_{B}}{\sigma}\right)-\Phi\left(\frac{Z_{CCC}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
CCC **TYPE:A** **TYPE:B** **TYPE:C** **TYPE:D** **TYPE:E** **TYPE:F**	$\Phi\left(\frac{\mu_A Z_{CCC}}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_{Def}}{\sigma}\right)\pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{\mu_A Z_{CCC}}{\sigma}\right)-\Phi\left(\frac{Z_{Def}}{\sigma}\right)\pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{\mu_A Z_{CCC}}{\sigma}\right)-\Phi\left(\frac{\mu_A Z_{Def}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{\mu_A Z_{CCC}}{\sigma}\right)-\Phi\left(\frac{Z_{Def}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{Z_{CCC}}{\sigma}\right)-\Phi\left(\frac{Z_{Def}}{\sigma}\right)\pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{Z_{CCC}}{\sigma}\right)-\Phi\left(\frac{Z_{Def}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$
Default **TYPE:A** **TYPE:B** **TYPE:C** **TYPE:D**	$\Phi\left(\frac{\mu_A Z_{Def}}{\sigma}\right)\pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{\mu_A Z_{Def}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{Z_{Def}}{\sigma}\right)\pm \sigma_A f\left(x,\mu_{company},\sigma_{company},\xi\right)$ $\Phi\left(\frac{Z_{Def}}{\sigma}\right)\pm f\left(x,\mu_{company},\sigma_{company},\xi\right)$

Existing Model (Laplace/Gauss – Hermite Default Rate Approximation):

The Laplace/Gauss – Hermite approximation of the likelihood Generalized Linear Mixed Models estimation is given by:

Default Rate Model:

$$PD_{k,t} = \Phi\left(\theta_{r(k)} + X_t\beta + Z_t\right)$$

With the following notation:

Φ is the standard normal cumulative distribution function

$r(k)$ is the rating of obligor k

$\theta_{r(k)}$ is an intercept for rating $r(k)$

X_t is an $1 \times p$ vector with the coefficients modeling the impact of the macro – economic variables on the PD

$Z_t \sim \Phi\left(0, \sigma^2\right)$ is a latent factor with variance σ^2. Latent factors can be correlated over time with correlation matrix $Corr\left(Z_1, Z_2, \ldots, Z_r\right) = C$.

Migration Model:

$$Pdown_{k,t} = \Phi\left(\theta_{r(k),d} + X_t\beta + Z_t\right)$$

$$Pup_{k,t} = 1 - \Phi\left(\theta_{r(k),d} + X_t\beta + Z_t\right)$$

Proposed Jameel's Models 22:

Up Migration Default Rate Models:

The Up Migration Default Rate Models of a Company under stress are given by:

TYPE A:

$$Pdown_{(k,t)stressed} = \Phi\left[\mu_A\left(\theta_{r(k),d} + X_t\beta + Z_t\right)\right] \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE B:

$$Pdown_{(k,t)stressed} = \Phi\left[\mu_A\left(\theta_{r(k),d} + X_t\beta + Z_t\right)\right] \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE C:

$$Pdown_{(k,t)stressed} = \Phi\left(\theta_{r(k),d} + X_t\beta + Z_t\right) \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

TYPE D:

$$Pdown_{(k,t)stressed} = \Phi\left(\theta_{r(k),d} + X_t\beta + Z_t\right) \pm f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$$

Down Migration Default Rate Models:

For example, the Down Migration Default Rate Models of a Company under stress using $M\ 22\ TYPE\ A$ are given by:

$$Pup_{(k,t)stressed} = 1 - \left[\Phi\left[\mu_A\left(\theta_{r(k),d} + X_t\beta + Z_t\right)\right] \pm \sigma_A f\left(x, \mu_{company}, \sigma_{company}, \xi\right)\right]$$

In similar way, we can find the Down Migration Default Rate Models of the remaining types.

RESULT AND DISCUSSION

Having presented the proposed Advanced Stressed Capital Requirements and Credit Rating Models and that the values of $\mu_{company}$, $\sigma_{company}$, μ_A and σ_A are computed using **EasyFit** and **QI Macros 2015 Softwares** then we are now ready to implement and present the Results of the proposed Models.

Example (reference to Black – Scholes – Merton (1973) Default Probability Model):

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained: $f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right) = 0.000073492$ (Log – Logistic (3P)), $\mu_A = 0.030383975$, and

$\sigma_A = 0.111414539$. Let $J = 0.464641$ then using the proposed Jameel's VII Models, we obtained the following table:

PROPOSED JAMEEL'S MODELS VII AND BLACK - SCHOLES FORMULA FOR CALCULATING PROBABILITY OF DEFAULT

M7 TYPE A+	M7 TYPE A-	M7 TYPE B+	M7 TYPE B-	M7 TYPE C+	M7 TYPE C-	M7 TYPE D+	M7 TYPE D-	BLACK -SCHOLES
0.494376251	0.494359875	0.494441555	0.494294571	0.321102471	0.321086095	0.321167775	0.321020791	0.321094283

Example (reference to Merton (1974) Recovery Rate Model):

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained:

$f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right) = 0.000073492$ (Log – Logistic (3P)), $\mu_A = 0.030383975$, and $\sigma_A = 0.111414539$. Let $A = 0.0508$, $D = 0.0627$, $T = 0.5$, $\mu_V = 0.00638311$, $d_1 = 0.464641$ and $d_2 = 0.3232196$ then using the proposed Jameel's VIII Models, we obtained the following table:

PROPOSED JAMEEL'S MODELS VIII AND BLACK - SCHOLES FORMULA FOR CALCULATING RECOVERY RATE

M8 TYPE A+	M8 TYPE A-	M8 TYPE B+	M8 TYPE B-	M8 TYPE C+	M8 TYPE C-	M8 TYPE D+	M8 TYPE D-	BLACK -SCHOLES RR
0.809997025	0.809980649	0.810062329	0.809915345	0.699202953	0.699186577	0.699268257	0.699121273	0.699194765

Example (reference to KMV – Merton):

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained:

$f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right) = 0.000073492$ (Log – Logistic (3P)), $\mu_A = 0.030383975$, and $\sigma_A = 0.111414539$. Let $DD = 0.3232196$ then using the proposed Jameel's IX Models, we obtained the following table:

PROPOSED JAMEEL'S MODELS IX AND KMV - MERTON FORMULA FOR CALCULATING PROBABILITY OF DEFAULT

M9 TYPE A+	M9 TYPE A-	M9 TYPE B+	M9 TYPE B-	M9 TYPE C+	M9 TYPE C-	M9 TYPE D+	M9 TYPE D-	KMV - MERTON
0.49609036	0.496073984	0.496155664	0.49600868	0.321102471	0.373256281	0.373337961	0.373190977	0.321094283

Example (reference to Naïve KMV – Merton):

Using the Chevron Corporation data extracted from yahoo finance from 2014 – 1991, we obtained:

$f\left(x, \mu_{Company}, \sigma_{Company}, \xi\right) = 0.000073492$ (Log – Logistic (3P)), $\mu_A = 0.030383975$, and $\sigma_A = 0.111414539$. Let $Naive\ DD = 0.53636121$ then using the proposed Jameel's X Models, we obtained the following table:

PROPOSED JAMEEL'S MODELS X AND NAIVE KMV - MERTON FORMULA FOR CALCULATING PROBABILITY OF DEFAULT

M10 TYPE A+	M10 TYPE A-	M10 TYPE B+	M10 TYPE B-	M10 TYPE C+	M10 TYPE C-	M10 TYPE D+	M10 TYPE D-	NAÏVE - KMV - MERTON
0.493506999	0.493490623	0.493572303	0.493425319	0.295862655	0.295846279	0.295927959	0.295780975	0.295854467

From the above tables, the **eight (8) proposed Jameel's Models** in each case gives much close approximation to that of **Original Black – Scholes, Merton, KMV – Merton and Naïve KMV - Merton** and interestingly captured **"fat – tail effect"** which is not being captured by the traditional once and has the ability to traces the trajectories of Past and Future **Economic and Financial Crises** given **accurate, valid and reasonable** estimations of the models' independent variables.

BANKING AND ECONOMIC APPLICATIONS OF DEFAULT PROBABILITIES AND RECOVERY RATE (LOSS GIVEN DEFAULT)

Formulas for calculating Risk Weighted Assets are derived from Merton (1974), single asset model to credit portfolios and Vasicek (2002) work. The initial version of the new Basel accord made an implicit assumption that Asset Correlation for all exposures is equal to 0.2. There has been quite some criticism about this assumption, and the Basel Committee decided to implement a revised formula for RWA where the correlation parameter depends on the estimated PD. The relationship between PD and Correlation can be described by the following expression:

Asset Correlation for all Exposures:

$$Correlation\left(R_{Stressed}\right) = 0.12 \times \frac{\left(1-\exp\left(-50\times PD_{Stressed}\right)\right)}{\left(1-\exp\left(-50\right)\right)} + 0.24\times\left[1-\frac{\left(1-\exp\left(-50\times PD_{Stressed}\right)\right)}{\left(1-\exp\left(-50\right)\right)}\right]$$

A very close approximation of this relationship is provided by the more simple expression:

$$Correlation\left(R_{Stressed}\right) = 0.12\times\left(1+\exp\left(-50\times PD_{Stressed}\right)\right)$$

Maturity Adjusted:

For firms with sales greater than Euro 500 Million in the advanced IRB approach, the Maturity Adjusted will be included according to the following factor:

$$MA_{Stressed} = \left(1+b_{Stressed}\times\left(M-2.5\right)\right)$$

With $b_{Stressed} = \left(0.11852-0.05478\times\ln\left(PD_{Stressed}\right)\right)^2$

Capital Requirement:

$$Stressed\ Capital\ Requirement\ \left(K_{Stressed}\right) = LGD_{Stressed}\times\Phi\left[\frac{\Phi^{-1}\left(PD_{Stressed}\right)}{\sqrt{1-R_{Stressed}}}+\sqrt{\frac{R_{Stressed}}{1-R_{Stressed}}}\times\Phi^{-1}\left(0.999\right)\right]\times\frac{\left(1+\left(M-2.5\right)\times b_{Stressed}\left(PD_{Stressed}\right)\right)}{\left(1-1.5\times b\left(PD_{Stressed}\right)\right)}$$

Risk Weighted Assets:

$$Stressed\ Risk-Weighted\ Assets\ \left(S\ RWA\right) = 12.5\times EAD\times K_{Stressed}$$

Regulatory Capital for Credit Risk:

$Stressed\ Regulatory\ Capital\ for\ Credit\ Risk = 8\%\times S\ RWA$ or in the other way round;

Worst – Case Default Rate:

Stressed Worst – Case Default Rate (S WCDR) used in the Basel II IRB approach is given by:

$$WCDR_{Stressed} = \Phi\left(\frac{\Phi^{-1}\left(PD_{Stressed}\right)+\sqrt{R_{Stressed}}\ \Phi^{-1}\left(0.999\right)}{\sqrt{1-R_{Stressed}}}\right)$$

$$S\ RWA = 12.5\times EAD_{Stressed}\times LGD_{Stressed}\times\left(WCDR-PD_{Stressed}\right)\times MA_{Stressed}$$

Unexpected Losses:

Also, the Stressed Unexpected Losses is given by:

$$UL_{Stressed} = LGD_{Stressed} \times \Phi\left(\frac{\Phi^{-1}\left(PD_{Stressed}\right) + \sqrt{R_{Stressed}}\ \Phi^{-1}(0.999)}{\sqrt{1 - R_{Stressed}}}\right) - LGD_{Stressed} \times PD_{Stressed}$$

$$UL_{Stressed} = LGD_{Stressed} \times \left(WCDR_{Stressed} - PD_{Stressed}\right)$$

Also, $UL_{Stressed} = \left(PD_{Stressed} - PD_{unconditional}\right) \times LGD_{Stressed} \times EAD_{Stressed}$

Relative Default Frequency:

The Relative Default Frequency is given by:

$$RDF_{Stressed} = \frac{PD_{Stressed}}{1 - PD_{Stressed}}$$

Capital Requirement for a Hedged Exposure Subject to Double Default Treatment (KDD):

The Capital Requirement for a Hedged Exposure Subject to Double Default Treatment (KDD) is calculated by multiplying K_0 as defined below by a multiplier depending on the PD of the protection provider $PD_{g\,(Stressed)}$.

$$K_{DD\,(Stressed)} = K_{0\,(Stressed)}\left(0.15 + 160 \times PD_{g\,(Stressed)}\right)$$

$K_{0\,(Stressed)}$ is calculated in the same way as a capital requirement for an unhedged corporate exposure, but using different parameters for LGD and the Maturity Adjustment.

$$K_{0(Stressed)} = LGD_{g\,(Stressed)}\left[\Phi\left(\frac{G\left(PD_{0(Stressed)}\right) + \sqrt{\rho_{os\,(Stressed)}} \times G(0.999)}{\sqrt{1 - \rho_{os\,(Stressed)}}}\right) - PD_{0\,(Stressed)}\right] \times \frac{\left(1 + (M - 2.5)\right)}{\left(1 - 1.5 \times b_{(Stressed)}\right)}$$

$$RWA_{DD(Stressed)} = K_{DD(Stressed)} \times 12.5 \times EAD_{g\,(Stressed)}$$

Where,

$\Phi(.)$ is the cumulative distribution function

$\Phi^{-1}(.)$ is the inverse cumulative distribution function

G(x) – inversion function to distribution function of normalized normal distribution

M is the effective (remaining) maturity

Finally, we can also use Risk Parameters to quantify Revolving Retail Exposures, other Retail Exposures, Supervisory Formula and many more under both Basel II and Basel III.

Example:

Consider the values obtained in the Proposed Jameel's Models VII (and Black – Scholes) to be **Chevron Corporation Probability of Defaut (not for an Option)** and Proposed Jameel's Models VIII (and Recovery Rate Black – Scholes). Let $EAD_{Stressed} = \$1,02000$ and $M = 3$ then we can Calculate:

(i) Stressed and Normal Asset Correlations for all exposures;
(ii) Stressed and Normal Capital Requirements;
(iii) Stressed and Normal Risk Weighted Assets;
(iv) Stressed and Normal Regulatory Capital for Credit Risk; and
(v) Stressed and Normal Unexpected Losses.

This can be seen using Microsoft **EXCEL** in the following table:

Stressed PDs, Rs, bs, RWAs, RCCRs and ULs:

FORMULAS	PD (STRESSED)	R (STRESSED)	b (STRESSED)	K (STRESSED)	$ RWA(STRESSED)	$ RCCR (STRESSED)	UL (STRESSED)	
M7 TYPE A+	0.494376251	0.12	0.024683625	0.119994617	1529931.364	122394.5091	0.071358583	
M7 TYPE A-	0.494359875	0.12	0.024684195	0.120001675	1530021.351	122401.7081	0.071366085	
M7 TYPE B+	0.494441555	0.12	0.024681351	0.119966466	1529572.442	122365.7953	0.071328667	
M7 TYPE B-	0.494294571	0.12	0.024686469	0.120029813	1530380.118	122430.4094	0.071396004	
M7 TYPE C+	0.321102471	0.120000013	0.032670492	0.131097555	1671493.829	133719.5063	0.126248224	
M7 TYPE C-	0.321086095	0.120000013	0.032671502	0.131098763	1671509.23	133720.7384	0.126255275	
M7 TYPE D+	0.321167775	0.120000013	0.032666465	0.131092731	1671432.324	133714.5859	0.126220106	
M7 TYPE D-	0.321020791	0.120000013	0.03267553	0.131103573	1671570.553	133725.6443	0.126283389	
BLACK -SCHOLES	PD (NORMAL)	R (NORMAL)	b (NORMAL)	K (NORMAL)	RWA(NORMAL)	RCCR (NORMAL)	UL (NORMAL)	
	0.321094283	0.120000013	0.032670997	0.131098159	1671501.53	133720.1224	0.12625175	
BLACK -SCHOLES RR	M8 TYPE A+	M8 TYPE A-	M8 TYPE B+	M8 TYPE B-	M8 TYPE C+	M8 TYPE C-	M8 TYPE D+	M8 TYPE D-
0.699194765	0.809997025	0.809980649	0.810062329	0.809915345	0.699202953	0.699186577	0.699268257	0.699121273
BLACK -SCHOLES LGD	LGD TYPE A+	LGD TYPE A-	LGD TYPE B+	LGD TYPE B-	LGD TYPE C+	LGD TYPE C-	LGD TYPE D+	LGD TYPE D-
0.300805235	0.190002975	0.190019351	0.189937671	0.190084655	0.300797047	0.300813423	0.300731743	0.300878727

From the above table, considering $(M7\ TYPE\ C-,\ LGD\ TYPE\ C-)$, the values for $R_{Stressed}$, $b_{Stressed}$, $K_{Stressed}$, $RWA_{Stressed}$, $RCCR_{Stressed}$ and $UL_{Stressed}$ are: **0.120000013, 0.032671502, 0.131098763, $1671509.23, $133720.7384 and 0.126255275** respectively, while that of $(M7\ TYPE\ D-,\ LGD\ TYPE\ D-)$ are: **0.120000013, 0.03267553, 0.131103573, $1671570.553, $133725.6443 and 0.126283389** which are all **HIGHER** or equal in values than corresponding Black – Scholes Formula (Normal) whose values are: **0.120000013, 0.032670997, 0.131098159, $1671501.53, $133720.1224 and 0.12625175.** Whereas in case of $M7\ TYPE\ A+$, $M7\ TYPE\ A-$, $M7\ TYPE\ B+$, $M7\ TYPE\ B-$, $M7\ TYPE\ C+,\ and\ M7\ TYPE\ D$, the values for $R_{Stressed}$, $b_{Stressed}$, $K_{Stressed}$, $RWA_{Stressed}$, $RCCR_{Stressed}$ and $UL_{Stressed}$ are all **LOWER or** equal than corresponding values in the case of Black – Scholes Formula (Normal).It would be recalled that Black – Scholes Formula suffered from the criticisms of **NORMALITY** assumption, that it can either **underestimates or overestimate Credit Risks**, therefore, from the foregoing, we can deduce the following:

(i) In case of Credit Risk **OVERESTIMATION** (stress period), we consider Jameel's Models: $(M7\ TYPE\ C-,\ LGD\ TYPE\ C-)$ and $(M7\ TYPE\ D-,\ LGD\ TYPE\ D-)$; whereas,

(ii) In case of Credit Risk **UNDERESTIMATION** (stress period), we consider Jameel's Models: $(M7\ TYPE\ A+,\ LGD\ TYPE\ A+)$, $(M7\ TYPE\ B+,\ LGD\ TYPE\ B+)$, $(M7\ TYPE\ B-,\ LGD\ TYPE\ B-)$, $(M7\ TYPE\ C+,\ LGD\ TYPE\ C+)$, and $(M7\ TYPE\ D+,\ LGD\ TYPE\ D+)$.

Similarly, we can treat the case of KMV – Merton and Naïve KMV – Merton in the same manner.

Proposed Jameel's Theorem 2:

Let $BS(X)$ be a **Black – Scholes** Model for calculating Probability of Default then there exists some Jameel's Probability Functions $J_1^2(X)$ and $J_2^2(X)$ such that $0 \le J_1^2(X) \le BS(X) \le J_2^2(X) \le 1$.

Proposed Jameel's Theorem 3:

Let $BSRR(X)$ be a **Black – Scholes Model** for calculating **Recovery Rates** then there exists some **Jameel's Probability Functions** $J_1^3(X)$ and $J_2^3(X)$ such that $0 \le J_1^3(X) \le BSRR(X) \le J_2^3(X) \le 1$.

Proposed Jameel's Theorem 4:

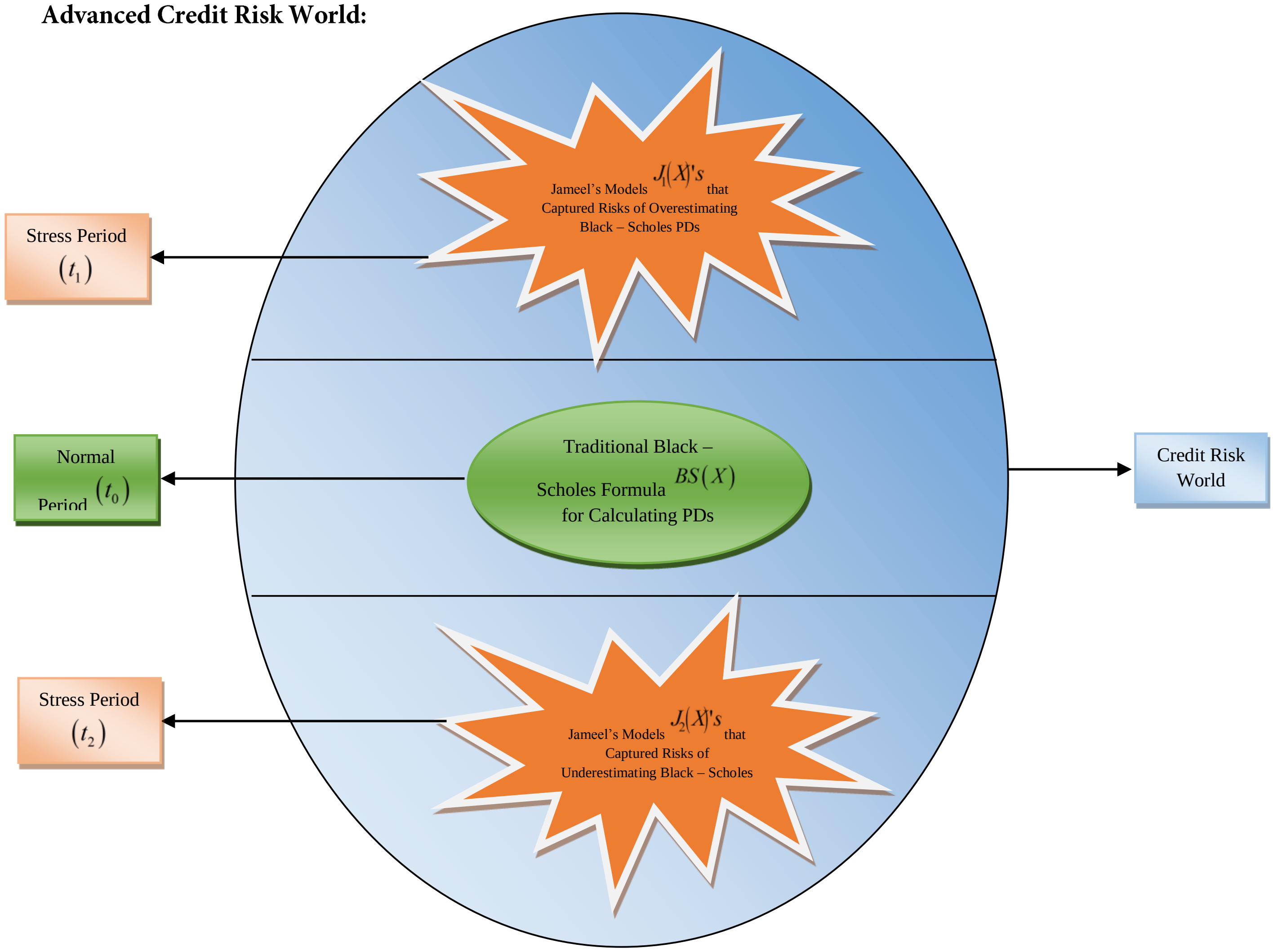
Let $KMVM(X)$ be a **KMV – Merton Model** for calculating Probability of Default then there exists some **Jameel's Probability Functions** $J_1^4(X)$ and $J_2^4(X)$ such that $0 \le J_1^4(X) \le KMVM(X) \le J_2^4(X) \le 1$.

Proposed Jameel's Theorem 5:

Let $NKMVM(X)$ be a **Naive KMV – Merton Model** for calculating Probability of Default then there exists some **Jameel's Probability Functions** $J_1^5(X)$ and $J_2^5(X)$ such that $0 \le J_1^5(X) \le NKMVM(X) \le J_2^5(X) \le 1$

\

Advanced Credit Risk World:



CONCLUSION

It would be critically observed **Jameel's models** reference to existing **traditional Logit and Probit Models** converges more **FASTER** than their counterparts, **Jameel's Models** reference to the **traditional Black Scholes, KMV – Merton and Naïve KMV – Merton** in which their convergence is **SLOW**, though the Author imposed some assumed numerical values.

For the sake of practitioners, it is believe that the existing Default Risk, Recovery Rate, Capital Requirements and Credit Risk Models out rightly underestimates (overestimates) Default Risks especially at the times of Economic and Financial Crises to the extent in which the popular Black Swans actors Nassim N. Taleb and Espen Gararder Hang (2011) wrote a paper entitled: ' Option Traders use (very) Sophisticated Heuristic, never the Black – Scholes – Merton Formula', cited that the formula is 'fragile to jumps and tail events' just because of its Normality Assumption thereby it is one of the major factors that contributed to the late 2007 – 2008 economic and financial crises (thanks to the Black – Scholes – Merton generous invention). In view of the above, all the proposed Jameel's Advanced Stressed Default Risk, Recovery Rate, Capital Requirements and Credit Risk Models presented in this paper, will be more robust, holistic and extraordinary, providing better approximations, increasing the probabilities of high losses and above all have the ability to precisely traces the trajectories of the past and future economic and financial crises related to Capital Requirements, Credit Ratings and Migrations, Basel II and III and general Credit Risk Management at large .

Finally, for the sake of future research direction, the models can be improved further to capture more vital information using more macroeconomic indicators and models' independent variables.

Nassim Nicholas Taleb et al (2009) stated that **"Black Swan events are almost impossible to predict.** Instead of perpetuating the illusion that we can anticipate the future, risk management should try to reduce the impact of the threats we don't understand."

CreditMetrics™ (1997) stated that "**We remind our readers that no amount of sophisticated analytics will replace experience and professional judgment in managing risks.** CreditMetrics™ is nothing more than a high-quality tool for the professional risk manager in the financial markets and is not a guarantee of specific results."

"If a seatbelt does not provide perfect protection, it still makes sense to wear one, it is better to wear a seatbelt than to not wear one". It is better off improving Probabilities of Default and Recovery Rates by incorporating fat – tail effects in our traditional Default Risk, Recovery Rate, Capital Requirements and Credit Risk Models than not.

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