

Research Article

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Sophisticated and Holistic Economic and Financial Meltdown Advanced Stressed Derivatives Pricing Models

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Abstract:

Extremistan is referred to as a Low – Probability, High – Impact events (Black Swan) occurs at the fourth quadrant.

The incompleteness of the Derivatives Pricing Models vis – a – vis traditional Normality Assumptions has catastrophically threaten the validity and robustness of the models to accurately capture the typical trajectories of the past and future economic and financial crises reference to the derivatives pricing.

Jamilu (2015), has attempted to stress the models by incorporating ONLY the probability distribution of the underlying stock returns (Log – Logistic 3P) and U.S. fundamental macroeconomic indicators.

The main aim of this paper is to holistically and more severely stress Jameel's Advanced Stressed Derivatives Pricing models by incorporating underlying stock returns (Log – Logistic 3P) and underlying strike prices (Cauchy) and U.S. fundamental macroeconomic indicators using Microsoft Corporation as a case study.

Finally, the extended versions were found efficiently working and have the ability to accurately and holistically capture the typical trajectories of the past and future economic and financial crises reference to Derivatives Pricing otherwise make the models more sophisticated and holistic.

KEYWORDS: Call, Put, Strike, Stress Test, Cauchy, Jameel.

INTRODUCTION:

The demarcation between the concepts of Mediocristan (Normal markets condition or events) and Extremistan (low – probability, high – impact or Black Swan events) as propagated by Nassim N. Taleb at el received serious attention in almost all the ramifications of knowledge, typically, in modern economic and financial world, particularly the Black – Scholes – Merton (1973) formula for both Call and Put Options. Many derivatives pricing models, bonds, stocks and bankruptcy prediction models were developed reference to them.

The ambiguity was that, Black – Scholes – Merton (1973) model has been seriously criticized as one of the fundamental factors that contributed to the late 2007 – 2008 financial crises because of its **NORMALITY ASSUMPTION**, this implies by extension, all the models derived and built vis – a – vis Black – Scholes – Merton (1973) model would have the same Normality assumption effects hence, they are not sophisticated and robust enough to withstand potential Black Swan events.

Jamilu (2015), incorporated fat – tailed effects by stressing Black – Scholes – Merton (1973) formula for both Call and Put Options so as to accurately capture potential Black Swan events using **ONLY** the probability distribution of the underlying stock returns (Log – Logistic 3P), the Geometric Return of the Arithmetic Means of the U.S. macroeconomic indicators plus research company underlying stock returns and the Geometric volatility of the volatilities of the U.S. macroeconomic indicators plus research company underlying stock returns in his paper entitled: **Global Economic and Financial Crises Advanced Stressed Derivatives Pricing Models**.

The main aim of this research paper is to make an attempt to holistically and more severely stress the proposed Jameel's (2015) models reference to **Black – Scholes – Merton (1973) Model**, **Garman - Kohlhagen (1983) Foreign Exchange Rates Options Price Model**, **Black (1976) Models** for pricing Caps, Floors and Swaptions so as to make them more sophisticated and robust to withstand potential Black Swan events.

MATERIAL AND METHODS

First, reference to the **Black – Scholes – Merton (1973)** Model, the price of Call and Put Options are given by

$$C(S, t) = N(d_1)S - N(d_2)Ke^{-r(T-t)} \quad \text{and}$$

$$P(S, t) = -\Phi(-d_1)S + \Phi(-d_2)Ke^{-r(T-t)} \quad \text{respectively.}$$

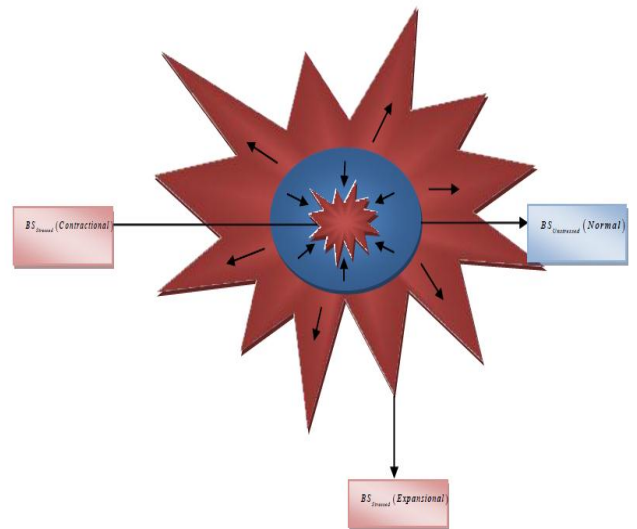
Jamilu (2015), consider **ONLY** the distribution of the underlying stock returns to stress the formulae for both Call and Put Options, however, **NEVER MIND** about the distribution of the **underlying strike prices** which is also another fundamental independent parameter to price both Call and Put options as shown in their formulae above. Also, the Strike Price is the **second major component** when determining (i) In the Money Options: $Max\{S - K, 0\}$ (ii) Out of the Money Options: $Max\{K - S, 0\}$ and (iii) At the Money Options: $Max\{S - K, 0\}$ such that $S = K$.

More so, after the Stock market Crash of 1987, the concepts of **VOLITILITY SMILE** and **SKEW** came into account. That is fat – tailed effects in the underlying assets distributions. This concept critically analyses the presence of **SKEW** and **HEAVY KUTOSIS** in the options strike prices over different volatilities. Furthermore, there is concrete evidence that the empirical returns distribution **DOES NOT** appear to be **GAUSSIAN**.

These are what makes it **MANDATORY** to incorporate the distribution of the underlying **STRIKE PRICES** in the extended versions of Jameel’s (2015) advanced Stressed Derivatives Pricing Models so as to makes them more sophisticated, Holistic and robust enough to significantly capture potential Black Swan events even **BETTER** than the previous ones.

The methodology adopted in this research work is to use Jameel’s Contractual and Expansional stress methods to **MORE SEVERELY** stress the traditional **Black – Scholes – Merton (1973) and its Greeks, Garman - Kohlhagen (1983) Foreign Exchange Options, Black (1976) for Caps, Floors and Swaptions Pricing Models**, using (a) Geometric Return of the Arithmetic Means of the U.S. macroeconomic indicators plus Microsoft Corporation underlying stock returns μ_s and the Geometric volatility of the volatilities of the U.S. macroeconomic indicators plus Microsoft Corporation underlying stock returns σ_s as well as the Best fitted fat – tailed probability distribution of the Microsoft Corporation underlying stock returns $f(s, \mu_s, \sigma_s, \xi)$ (b) Geometric Return of the Arithmetic Means of the U.S. macroeconomic indicators plus Microsoft Corporation underlying strike prices μ_k and the Geometric volatility of the volatilities of the U.S.

macroeconomic indicators plus Microsoft Corporation underlying strike prices σ_k as well as the Best fitted fat – tailed probability distribution of the Microsoft Corporation underlying strike prices $f(k, \mu_k, \sigma_k, \pi)$ (c) Geometric Return of the Arithmetic Means of the U.S. macroeconomic indicators plus Microsoft Corporation underlying stock returns and underlying strike prices μ_{s_k} and the Geometric volatility of the volatilities of the U.S. macroeconomic indicators plus Microsoft Corporation underlying stock returns and underlying strike prices σ_{s_k} :



First Proposed Jameel’s Models I:

The proposed models considering **Black – Scholes – Merton (1973) Call Option Price** are given by:

$$C(S, t)_{Stressed} = S [\Phi(\mu_s, \mu_k, d_1) \pm \sigma_s \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_k \cdot f(k, \mu_k, \sigma_k, \pi)] - K [\Phi(\mu_s, \mu_k, d_2) \pm \sigma_s \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_k \cdot f(k, \mu_k, \sigma_k, \pi)] \cdot e^{-r(T-t)}$$

The first Type above is the general (initial) proposal not necessarily positive prices (but could also be negative values); however, using Jameel’s Contractual and Expansional Stress Methods, we can have the following possible **COMBINATION** of **TERMS** and **SIGNS**.

Combination of Terms and Signs:

Recall that $C_r^n = n! / (n - r)! r!$ then we have $C_1^6 + C_2^6 + C_3^6 + C_4^6 + C_5^6 + C_6^6 = 63$ combination of terms using the general form above and $C_1^8 + C_2^8 + C_3^8 + C_4^8 + C_5^8 + C_6^8 + C_7^8 + C_8^8 = 225$ combination of Signs as follows:

Therefore, we have to further check 4 and 162 remaining combination of Terms and Signs respectively.

Where, $P(S, t) = -\Phi(-d_1)S + \Phi(-d_2)Ke^{-r(T-t)}$.

Second Proposed Jameel's Models I:

The proposed models considering **Black – Scholes – Merton (1973) Call Option Price** are given by:

$$C(S, t)_{Stressed} = S \left[\Phi(\mu_{SK}d_1) \pm \sigma_{SK}f(s, \mu_S, \sigma_S, \xi) \pm \sigma_{SK}f(k, \mu_K, \sigma_K, \pi) \right] - K \left[\Phi(\mu_{SK}d_2) \pm \sigma_{SK}f(s, \mu_S, \sigma_S, \xi) \pm \sigma_{SK}f(k, \mu_K, \sigma_K, \pi) \right] e^{-r(T-t)}$$

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both **CALL** and **PUT** options as in the case of first Proposed Jameel's I Models shown in the table above.

Also, the **GREEKS** of the both **CALL** and **PUT** options can be found in similar ways and patterns.

Therefore, in this section, the author will treat the proposed Jameel's models II to VIII as he treated the First and Second proposed Jameel's models I above. Similarly in the case of Result and Discussion section.

Proposed Jameel's Models II:

The proposed models considering **Garman - Kohlhagen (1983) Foreign Exchange Rates Options Price** are given by:

$$C(S_0, t)_{Stressed} = \left[F \left[\Phi(\mu_S, \mu_K, d_1) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] - K \left[\Phi(\mu_S, \mu_K, d_2) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] \right] e^{-r(T-t)}$$

And/or

$$C(S_0, t)_{Stressed} = \left[F \left[\Phi(\mu_{SK}d_1) \pm \sigma_{SK}f(s, \mu_S, \sigma_S, \xi) \pm \sigma_{SK}f(k, \mu_K, \sigma_K, \pi) \right] - K \left[\Phi(\mu_{SK}d_2) \pm \sigma_{SK}f(s, \mu_S, \sigma_S, \xi) \pm \sigma_{SK}f(k, \mu_K, \sigma_K, \pi) \right] \right] e^{-r(T-t)}$$

and that of **PUT** are given by:

$$P(S_0, t)_{Stressed} = \left[-F \left[\Phi(-\mu_S, \mu_K, d_1) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] + K \left[\Phi(-\mu_S, \mu_K, d_2) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] \right] e^{-r(T-t)}$$

And/or

$$C(S_0, t)_{Stressed} = \left[-F \left[\Phi(-\mu_{SK}d_1) \pm \sigma_{SK}f(s, \mu_S, \sigma_S, \xi) \pm \sigma_{SK}f(k, \mu_K, \sigma_K, \pi) \right] + K \left[\Phi(-\mu_{SK}d_2) \pm \sigma_{SK}f(s, \mu_S, \sigma_S, \xi) \pm \sigma_{SK}f(k, \mu_K, \sigma_K, \pi) \right] \right] e^{-r(T-t)}$$

Where, $C(S_0, t) = e^{-r(T-t)} (F\Phi(d_1) - K\Phi(d_2))$ and

$$P(S_0, t) = e^{-r(T-t)} (K\Phi(-d_2) - F\Phi(-d_1)).$$

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both **CALL** and **PUT** options as in **ALL** the cases as shown in the tables above.

Proposed Jameel's Models III:

The proposed models considering **Black (1976) for CAPS Price** are given by:

$$Cap(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[F(t, T_{i-1}, T_i) \left[\Phi(\mu_{SK}d_1) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] - K \left[\Phi(\mu_{SK}(d_1 - \sigma\sqrt{T_{i-1}-t})) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] \right]$$

And/or

$$Cap(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[F(t, T_{i-1}, T_i) \left[\Phi(\mu_{SK}d_1) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] - K \left[\Phi(\mu_{SK}(d_1 - \sigma\sqrt{T_{i-1}-t})) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] \right]$$

and that of **FLOORLET** are given by:

$$Floor(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[-F(t, T_{i-1}, T_i) \left[\Phi(-\mu_{SK}d_1) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] + K \left[\Phi(\mu_{SK}(-d_1 + \sigma\sqrt{T_{i-1}-t})) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] \right]$$

And/or

$$Floor(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[-F(t, T_{i-1}, T_i) \left[\Phi(-\mu_{SK}d_1) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] + K \left[\Phi(\mu_{SK}(-d_1 + \sigma\sqrt{T_{i-1}-t})) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] \right]$$

Where,

$$Cap(t) = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[F(t, T_{i-1}, T_i) \Phi(d_i) - E \times \Phi(d_i - \sigma\sqrt{T_{i-1}-t}) \right]$$

and.

$$Floor(t) = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[-F(t, T_{i-1}, T_i) \Phi(-d_i) + E \times \Phi(-d_i + \sigma\sqrt{T_{i-1}-t}) \right]$$

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both **CAPS** and **FLOORS** as in **ALL** the cases as shown in the tables above.

Proposed Jameel's Models IV:

The proposed models considering **Black (1976) for PAYER SWAPTIONS Prices** are given by:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left[F_i(t) \times \left[\Phi(\mu_{SK}d) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] - K \left[\Phi(\mu_{SK}(d - \sigma\sqrt{T_0-t})) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] \right] \times D(t, T_i)$$

And/or

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left[F_i(t) \times \left[\Phi(\mu_{SK}d) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] - K \left[\Phi(\mu_{SK}(d - \sigma\sqrt{T_0-t})) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] \right] \times D(t, T_i)$$

and that of **RECEIVER SWAPTIONS Prices** are given by:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left[-F_i(t) \times \left[\Phi(-\mu_{SK}d) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] + K \left[\Phi(\mu_{SK}(-d + \sigma\sqrt{T_0-t})) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] \right] \times D(t, T_i)$$

And/or

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left[-F_i(t) \times \left[\Phi(-\mu_{SK}d) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] + K \left[\Phi(\mu_{SK}(-d + \sigma\sqrt{T_0-t})) \pm \sigma_S f(s, \mu_S, \sigma_S, \xi) \pm \sigma_K f(k, \mu_K, \sigma_K, \pi) \right] \right] \times D(t, T_i)$$

Where, Payer Swaption is given by:

$$Swaption(t) = \sigma \times M \times \sum_{i=1}^n \left(F_s(t) \times \Phi(d) - F \times \Phi(d - \sigma\sqrt{T_0-t}) \right) \times D(t, T_i)$$

and

Receiver Swaption is given by:

$$Swaption(t) = \sigma \times M \times \sum_{i=1}^n \left(-F_s(t) \times \Phi(-d) + F \times \Phi(-d + \sigma\sqrt{T_0-t}) \right) \times D(t, T_i)$$

Similarly in this case, we can find the 63 combination of terms and 225 combinations of signs for the both **PAYER SWAPTION** and **RECEIVER SWAPTION** as in **ALL** the cases as shown in the tables above.

RECOVERY RATES AND DEFAULT PROBABILITIES OF THE EXTENDED VERSIONS

Proposed Jameel’s Models V:

The proposed models considering **Black – Scholes – Merton (1973) Default Probability Formula** are given by:

$$PD_{Stressed} = \Phi(-\mu_s \cdot \mu_k \cdot J) \pm \sigma_s \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_k \cdot f(k, \mu_k, \sigma_k, \pi)$$

And/or

$$PD_{Stressed} = \Phi(-\mu_{SK} \cdot J) \pm \sigma_{SK} \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_{SK} \cdot f(k, \mu_k, \sigma_k, \pi)$$

We have $C_1^3 + C_2^3 + C_3^3 = 7$ combination of terms and $C_1^4 + C_2^4 + C_3^4 + C_4^4 = 15$ combination of Signs of this

nature. Where,
$$PD = \Phi \left[\frac{- \left(\ln \frac{A_t}{L} + \mu_v - \frac{\sigma_v^2}{2} (T - t) \right)}{\sigma_v \sqrt{T - t}} \right]$$

with $J = \frac{\left(\ln \frac{A_t}{L} + \mu_v - \frac{\sigma_v^2}{2} (T - t) \right)}{\sigma_v \sqrt{T - t}}$ then

$$PD = \Phi(-J)$$

Proposed Jameel’s Models VI:

The proposed models considering **Black – Scholes – Merton (1973) Recovery Rate Formula** are given by:

$$RR_{Stressed} = \frac{A_0}{D} \exp(\mu_v T) \frac{\Phi(-\mu_s \cdot \mu_k \cdot d_1)}{\Phi(-\mu_s \cdot \mu_k \cdot d_2)} \pm \sigma_s \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_k \cdot f(k, \mu_k, \sigma_k, \pi)$$

And/or

$$RR_{Stressed} = \frac{A_0}{D} \exp(\mu_v T) \frac{\Phi(-\mu_{SK} \cdot d_1)}{\Phi(-\mu_{SK} \cdot d_2)} \pm \sigma_{SK} \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_{SK} \cdot f(k, \mu_k, \sigma_k, \pi)$$

We have $C_1^3 + C_2^3 + C_3^3 = 7$ combination of terms and $C_1^4 + C_2^4 + C_3^4 + C_4^4 = 15$ combination of Signs of this

nature. Where,
$$RR = \frac{A_0}{D} \exp(\mu_v T) \frac{\Phi(-d_1)}{\Phi(-d_2)}$$

$$d_1 = \frac{\left(\ln \frac{A_t}{L} + \mu_v - \frac{\sigma_v^2}{2} T \right)}{\sigma_v \sqrt{T}} \text{ and } d_2 = d_1 - \sigma_v T$$

Proposed Jameel’s Models VII:

The proposed models considering **KMV – Merton** are given by:

$$PD_{Stressed} = \Phi(-\mu_s \cdot \mu_k \cdot DD) \pm \sigma_s \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_k \cdot f(k, \mu_k, \sigma_k, \pi)$$

And/or

$$PD_{Stressed} = \Phi(-\mu_{SK} \cdot DD) \pm \sigma_{SK} \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_{SK} \cdot f(k, \mu_k, \sigma_k, \pi)$$

We have $C_1^3 + C_2^3 + C_3^3 = 7$ combination of terms and $C_1^4 + C_2^4 + C_3^4 + C_4^4 = 15$ combination of Signs of this nature. Where,

$$\pi K M V = \Phi \left[\frac{\left(\ln \left(\frac{V}{F} \right) + (\mu - 0.5 \sigma_v^2) T \right)}{\sigma_v \sqrt{T}} \right] \text{ with}$$

$$D D = \frac{\ln \left(\frac{V}{F} \right) + (\mu - 0.5 \sigma_v^2) T}{\sigma_v \sqrt{T}}$$

$$\pi K M V = \Phi(-D D)$$

Proposed Jameel’s Models VIII:

The proposed models considering **Naive KMV – Merton Alternative** are given by:

$$PD_{Stressed} = \Phi(-\mu_s \cdot \mu_k \cdot NaiveDD) \pm \sigma_s \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_k \cdot f(k, \mu_k, \sigma_k, \pi)$$

And/or

$$PD_{Stressed} = \Phi(-\mu_{SK} \cdot NaiveDD) \pm \sigma_{SK} \cdot f(s, \mu_s, \sigma_s, \xi) \pm \sigma_{SK} \cdot f(k, \mu_k, \sigma_k, \pi)$$

We have $C_1^3 + C_2^3 + C_3^3 = 7$ combination of terms and $C_1^4 + C_2^4 + C_3^4 + C_4^4 = 15$ combination of Signs of this nature. Where,

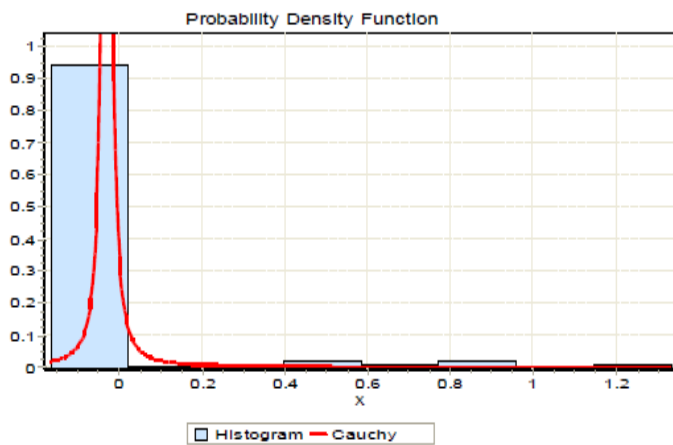
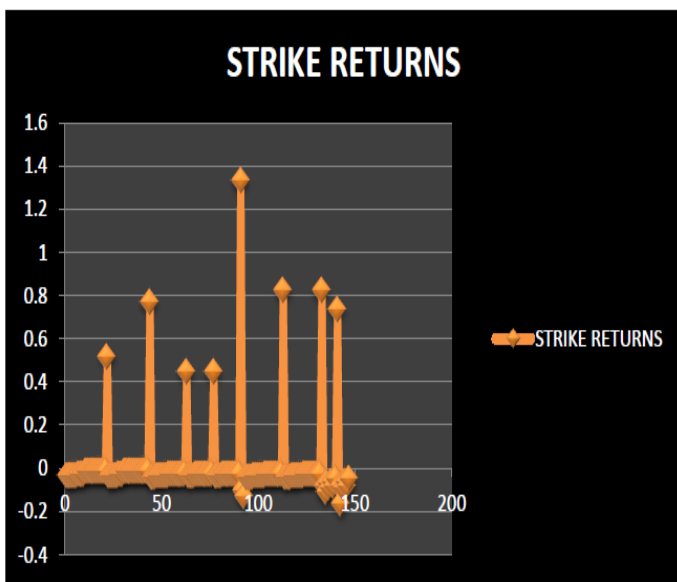
$$Naive D D = \frac{\ln \left[\frac{(E + F)}{F} \right] + (r_{u-1} - 0.5 Naive \sigma_v^2) T}{Naive \sigma_v \sqrt{T}}$$

with $\pi_{Naive} = \Phi(-Naive D D)$.

$$Naive \sigma_D = 0.05 + 0.25 \sigma_E \text{ and } Naive \mu = r_{u-1}$$

RESULT AND DISCUSSION

Using **Microsoft Corporation (MSFT)** options underlying Strike Prices (Returns) for the period of 148 Months (as an example). It can be observed from the graph of the relationship, the underlying strike prices (returns) skewed with heavy kurtosis as shown below: By running the goodness of fit test of the underlying Strike Prices (Returns) over 148 Months using Easyfit Software, we have the following summary result:



Distribution	Kolmogorov Smirnov	Anderson Darling	Chi-Squared	Remark
Cauchy	1	1	1	1 st
Log - Logistic 3P	2	4	2	2 nd
Dagum 4P	3	3	3	3 rd

The **Best fitted fat - tailed Probability Distribution Function** using Jameel's criterion is **Cauchy** which is the probability distribution of the **underlying Strike Prices (Returns)** for the period of 148 Months and is given by:

$$f(x; \mu_{underlying}, \sigma_{underlying}, \pi) = \left[\pi \sigma \left(1 + \left(\frac{x - \mu}{\sigma} \right)^2 \right) \right]^{-1}$$

Now, having presented the proposed extended versions of Jameel's Advanced Stressed Derivatives Pricing Models, computed the values of $\mu_{company}$, $\sigma_{company}$, μ_s , σ_s , μ_K , σ_K , μ_{SK} , and σ_{SK} and obtained the Best fitted fat -

tailed probability distribution of the underlying Strike Returns $f(k; \mu_K, \sigma_K, \pi)$ (Cauchy) then we are ready to implement and discuss the Results of the proposed Jameel's Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models as follows.

Example 1: Consider an example of **Microsoft Corporation (MSFT)** option with a term of six months (0.5 years). The current stock price of **Microsoft Corporation (MSFT)** is \$48.14 and the strike of the option is \$49.39. The risk-free rate is 3.92% p.a. The volatility of the stock is 2.2041976% p.a. With the above strike prices of 148 months: $\mu_{strike} = 0.008836$, $\sigma_{strike} = 0.188051$, and $f(k; \mu_K, \sigma_K, \pi) = 1.624231$ (Cauchy) for the current period. Note that, unlike Probability, Probability Distributions Function can take values **GREATER THAN ONE** at extreme cases since its define as Probability **PER UNIT VALUE** of a Random Variable, but the integral of this distribution function taken with respect to this value must be exactly equal 1. What is the values of both **CALL** and **PUT** the options using:

- (1) Black-Scholes - Merton (1973) Model; and
- (2) Proposed Jameel's Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models reference to Black-Scholes - Merton (1973) Model?

Using the data of **Microsoft Corporation (MSFT)** extracted from **yahoo finance from 2014 - 1991**, we obtained:

$$f(x, \mu_{underlying}, \sigma_{underlying}, \xi) = 0.00000000123523 \text{ (Log - Logistic (3P))}, \mu_s = 0.031886784, \sigma_s = 0.117906073, \mu_K = 0.032829086, \sigma_K = 0.124525303, \mu_{SK} = 0.028046277, \text{ and } \sigma_{SK} = 0.123540719 \text{ (data available)}, K = \$49.39, S = \$48.14, \sigma = 0.022041976, r = 0.0392, T = 0.5 \text{ and } t = 0.$$

$$\text{Recall that } d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T - t)}{\sigma\sqrt{T - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T - t} \text{ then}$$

$$d_1 = \frac{\ln\left(\frac{48.14}{49.39}\right) + \left(0.0392 + \frac{0.022041976^2}{2}\right)(0.5 - 0)}{0.022041976\sqrt{0.5 - 0}} = -0.18969$$

$$d_2 = -0.18969 - 0.022041976\sqrt{0.5} = -0.20528$$

Therefore, $d_1 = -0.18969$ and $d_2 = -0.20528$. Using the **Microsoft EXCEL**, consider the following tables:

Call Option Black – Scholes – Merton (1973) and Jameel’s Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models:

TYPES (CALL)	TYPE n+	BLACK -SCHOLES	TYPE n-	TYPE n+	BLACK -SCHOLES	TYPE n-	TYPE n+	BLACK -SCHOLES	TYPE n-	TYPE n+	BLACK -SCHOLES	TYPE n-
1, 2, 3, 4	-0.618622475	0.171627934	0.327918129	-0.301642368	0.171627934	0.644898236	0.112693806	0.171627934	0.230562062	-4.236090898	0.171627934	-3.289550294
5, 6, 7, 8	-0.301642315	0.171627934	0.644898183	68.56648089	0.171627934	-68.22322502	3.315826003	0.171627934	4.262366712	-68.7554295	0.171627934	69.09868537
9, 10, 11, 12	-72.68987798	0.171627934	65.16423679	-4.236090845	0.171627934	-3.289550347	68.56648094	0.171627934	-68.22322507	64.63203236	0.171627934	-72.15767355
13, 14, 15, 16	68.56648084	0.171627934	-68.22322497	-68.7554294	0.171627934	69.09868527	0.112693806	0.171627934	0.230562062	-0.301642315	0.171627934	0.644898183
17, 18, 19, 20	72.18394931	0.171627934	-64.6057566	-4.236090951	0.171627934	-3.289550242	-65.13796103	0.171627934	72.71615374	64.63203236	0.171627934	-72.15767355
21, 22, 23, 24	-65.13796108	0.171627934	72.7161538	64.63203241	0.171627934	-72.1576736	-72.68987803	0.171627934	65.16423684	64.63203231	0.171627934	-72.1576735
25, 26, 27, 28	-72.68987793	0.171627934	65.16423674	-65.13796098	0.171627934	72.71615369	-68.75542945	0.171627934	69.09868532	72.18394926	0.171627934	-64.60575655
29, 30, 31, 32	68.24950078	0.171627934	-68.54020513	72.18394937	0.171627934	-64.60575665	-0.618622422	0.171627934	0.327918076	-3.821754724	0.171627934	-3.703886468
33, 34, 35, 36	0.112693859	0.171627934	0.230562009	3.315826055	0.171627934	4.262366659	-0.618622528	0.171627934	0.327918181	-0.618622422	0.171627934	0.327918076
37, 38, 39, 40	-0.618622528	0.171627934	0.327918181	-3.821754672	0.171627934	-3.703886521	68.24950084	0.171627934	-68.54020518	72.18394931	0.171627934	-64.6057566
41, 42, 43, 44	3.730162177	0.171627934	3.730162177	-69.07240961	0.171627934	68.78170527	-69.07240951	0.171627934	68.78170516	68.24950073	0.171627934	-68.54020508
45, 46, 47, 48	68.24950084	0.171627934	-68.54020518	-0.618622475	0.171627934	0.327918129	-0.204286301	0.171627934	-0.086418045	-0.204286301	0.171627934	-0.086418045
49, 50, 51, 52	-72.68987798	0.171627934	-72.68987798	3.730162282	0.171627934	3.848030432	64.63203236	0.171627934	-72.15767355	3.730162229	0.171627934	3.848030485
53, 54, 55, 56	-3.821754724	0.171627934	-3.703886468	-0.204286248	0.171627934	-0.086418098	68.24950078	0.171627934	-68.54020513	-0.204286301	0.171627934	-0.086418045
57, 58, 59	3.730162177	0.171627934	3.848030538	-65.13796103	0.171627934	72.71615374	0.112693754	0.171627934	0.230562115		0.171627934	

Based on the above CALL option table: Jameel’s models **3, 33, and 59** are **EXTREMELY** recommended at the times of Economic and Financial **MELTDOWN** (recoveries and recessions stress periods) while, **LEFT** of models **1, 2, 5, 16, 31, 35, 36** are partially **HIGHER** values reference to the **BSM’s** price, nevertheless, they are also useful. Jameel’s models **7, 34, 41, 50, 52 and 57** are **EXTREMELY** higher values considering **BSM** but could be useful in other market conditions.

Call Option Black – Scholes – Merton (1973), First and Second Jameel’s Proposed Models reference to BSM

CALL	1ST PROPOSED MODEL CLASS (+)	1ST PROPOSED MODEL CLASS (-)	BLACK -SCHOLES	2ND PROPOSED MODEL CLASS (+)	2ND PROPOSED MODEL CLASS (-)
M I, II TYPE 3	0.112693806	0.230562062		0.11315978	0.230096088
M I, II TYPE 33	0.112693859	0.230562009	0.171627934	0.113159833	0.230096035
M I, II TYPE 59	0.112693754	0.230562115		0.112693754	0.23009614

The above table summarized the comparisons between **FIRST** and **SECOND** proposed Jameel’s models I. comparing the columns of 1st proposed model class (+) and 2nd proposed model class (+), they ultimately approximates one another with a very strong positive correlation, similarly, 1st proposed model class (-) and 2nd proposed model class (-) with their values sufficiently in between **BSM** Price. These conclude that both **FIRST** and **SECOND** proposed Jameel’s models are extremely recommended to be used at the times of Economic and Financial **MELTDOWN** (recoveries and recessions stress periods).

Put Option Black – Scholes – Merton (1973) and Jameel’s Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models:

TYPES (PUT)	TYPE n+	BLACK -SCHOLES	TYPE n-	TYPE n+	BLACK -SCHOLES	TYPE n-	TYPE n+	BLACK -SCHOLES	TYPE n-	TYPE n+	BLACK -SCHOLES	TYPE n-
1, 2, 3, 4	0.619299282	0.463009087	-0.327241322	0.936279389	0.463009087	-0.010261215	0.521943215	0.463009087	0.404074959	-2.998169141	0.463009087	-3.944709745
5, 6, 7, 8	0.936279336	0.463009087	-0.010261162	-67.93184387	0.463009087	68.85786204	4.553747865	0.463009087	3.607207156	69.39006653	0.463009087	-68.46404835
9, 10, 11, 12	65.45561794	0.463009087	-72.39849683	-2.998169194	0.463009087	-3.944709692	-67.93184392	0.463009087	68.8578621	-71.8662924	0.463009087	64.92341351
13, 14, 15, 16	-67.93184382	0.463009087	68.85786199	69.39006642	0.463009087	-68.46404825	0.521943215	0.463009087	0.404074959	0.936279336	0.463009087	-0.010261162
17, 18, 19, 20	-64.31437545	0.463009087	72.47533047	-2.998169089	0.463009087	-3.944709798	73.0075349	0.463009087	-64.84657988	-71.8662924	0.463009087	64.92341351
21, 22, 23, 24	73.00753495	0.463009087	-64.84657993	-71.86629245	0.463009087	64.92341357	65.455618	0.463009087	-72.39849688	-71.86629235	0.463009087	64.92341346
25, 26, 27, 28	65.45561789	0.463009087	-72.39849678	73.00753484	0.463009087	-64.84657982	69.39006647	0.463009087	-68.4640483	-64.31437539	0.463009087	72.47533041
29, 30, 31, 32	-68.24882398	0.463009087	68.54088194	-64.3143755	0.463009087	72.47533052	0.619299229	0.463009087	-0.327241269	-3.412505315	0.463009087	-3.530373571
33, 34, 35, 36	0.521943163	0.463009087	0.404075012	4.553747812	0.463009087	3.607207209	0.619299334	0.463009087	-0.327241375	0.619299229	0.463009087	-0.327241269
37, 38, 39, 40	0.619299334	0.463009087	-0.327241375	-3.412505368	0.463009087	-3.530373518	-68.24882403	0.463009087	68.54088199	-64.31437545	0.463009087	72.47533047
41, 42, 43, 44	4.139411691	0.463009087	4.02154333	69.07308642	0.463009087	-68.78102846	69.07308631	0.463009087	-68.78102835	-68.24882392	0.463009087	68.54088188
45, 46, 47, 48	-68.24882403	0.463009087	68.54088199	0.619299282	0.463009087	-0.327241322	0.204963108	0.463009087	0.087094852	0.204963108	0.463009087	0.087094852
49, 50, 51, 52	65.45561794	0.463009087	-72.39849683	4.139411586	0.463009087	4.021543435	-71.8662924	0.463009087	64.92341351	4.139411638	0.463009087	4.021543383
53, 54, 55, 56	-3.412505315	0.463009087	-3.530373571	0.204963055	0.463009087	0.087094905	-68.24882398	0.463009087	68.54088194	0.204963108	0.463009087	0.087094852
57, 58, 59	4.139411691	0.463009087	4.02154333	73.0075349	0.463009087	-64.84657988	0.521943268	0.463009087	0.404074907		0.463009087	

Based on the above PUT option table: Jameel’s models **3, 33, 48, 54 and 59** are **EXTREMELY** recommended at the times of Economic and Financial **MELTDOWN** (recoveries and recessions stress periods) while, **LEFT** of models **1, 2, 5, 16, 31, 35, 36** and **47** are partially **HIGHER** values reference to the **BSM’s price**, nevertheless, they are also useful Jameel’s models **7, 34, 41, 50, 52 and 57** are **EXTREMELY** higher values considering **BSM** but could be useful in other market conditions.

Put Option Black – Scholes – Merton (1973), First and Second Jameel’s Proposed Models reference to BSM

PUT	1ST PROPOSED MODEL CLASS (+)	1ST PROPOSED MODEL CLASS (-)	BLACK -SCHOLES	2ND PROPOSED MODEL CLASS (+)	2ND PROPOSED MODEL CLASS (-)
M I, II TYPE 3	0.521943215	0.404074959		0.521477241	0.404540933
M I, II TYPE 33	0.521943163	0.404075012	0.463009087	0.521477188	0.404540986
M I, II TYPE 59	0.521943268	0.404074907		0.521943268	0.404540881

The above table summarized the comparisons between **FIRST** and **SECOND** proposed Jameel’s models I. comparing the columns of 1st proposed model class (+) and 2nd proposed model class (+), they ultimately approximates one another with a very strong positive correlation, similarly, 1st proposed model class (-) and 2nd proposed model class (-) with their values sufficiently in between **BSM Price**. These conclude that both **FIRST** and **SECOND** proposed Jameel’s models are extremely recommended to be used at the times of Economic and Financial **MELTDOWN** (recoveries and recessions stress periods).

Generally, by alternating the **TERMS** and **SIGNS (up to 163 and 225)** of all the proposed Jameel's Models, we can observe more values that will approximate the existing derivatives models presented in this paper and can be used at the times of Economic and Financial **MELTDOWN** (recoveries and recessions stress periods).

Proposed Theorem (Jameel's MMM 1):

Let M_{Normal} be a **Black – Scholes – Merton (1973) for pricing options, Garman - Kohlhagen (1983) Foreign Exchange Rates Options, Black (1976) for pricing Caps, Floors, Payer and Receiver Options**, let $a, b \in R^+$ such that $a < b$, then using Jameel's Contractual and Expansional stress methods, there exists some Jameel's Sophisticated and Holistic Advanced Stressed Derivatives Pricing Models $J_1(x, \mu, \sigma, \xi)$ and $J_2(x, \mu, \sigma, \xi)$ such that $a < J_1(x, \mu, \sigma, \xi) \leq M_{Normal} \leq J_2(x, \mu, \sigma, \xi) < b$, depending on the derivative in question.

Proposed Theorem (Jameel's MMM 2):

Let $P_i = \Phi(d_i)$, $i = 1, 2, \dots, n$ be an independent probabilities components of traditional derivatives pricing model M_{Normal} whose probabilities is in this form. Let x and $f(x, \mu, \sigma, \xi)$ be the underlying stock return and fat – tailed probability distribution of the underlying stock returns of company **A** respectively. Let y_1, y_2, \dots, y_n be the underlying returns of Company **A** (traditional derivative pricing model M_{Normal}) fundamental independent (explained) variables. Let $f_1(y_1, \mu, \sigma, \pi)$, $f_2(y_2, \mu, \sigma, \pi), \dots, f_n(y_n, \mu, \sigma, \pi)$ be fat – tailed probability distribution of the underlying independent variables of company **A** (traditional derivative pricing model M_{Normal}). Let $(\mu_{y_1}, \mu_{y_2}, \dots, \mu_{y_n})$ and $(\sigma_{y_1}, \sigma_{y_2}, \dots, \sigma_{y_n})$ be vectors of Geometric Return of the Arithmetic Means of the macroeconomic indicators plus research company underlying stock returns (and independent variables returns) and the Geometric volatility of the volatilities of the macroeconomic indicators plus research company underlying stock returns (and independent variables returns) respectively. Define μ_s and σ_s as above then $P_i = \Phi(d_i)$, $i = 1, 2, \dots, n$ can be contractionally and expansionally stressed using Jameel's stress methods as:

$$P_{i(Stressed)} = \Phi(\mu_{y_1, \mu_{y_2}, \dots, \mu_{y_n}, d_i) \pm \sigma_s \cdot f(x, \mu, \sigma, \xi) \pm \sigma_{y_1} \cdot f_1(y_1, \mu, \sigma, \pi) \pm \sigma_{y_2} \cdot f_2(y_2, \mu, \sigma, \pi) \pm \dots \pm \sigma_{y_n} \cdot f_n(y_n, \mu, \sigma, \pi)$$

Optimum at:

$$P_{i(Stressed)} = \Phi(d_i) \pm \sigma_s \cdot f(x, \mu, \sigma, \xi) \pm \sigma_{y_1} \cdot f_1(y_1, \mu, \sigma, \pi) \pm \sigma_{y_2} \cdot f_2(y_2, \mu, \sigma, \pi) \pm \dots \pm \sigma_{y_n} \cdot f_n(y_n, \mu, \sigma, \pi)$$

for some $\sigma's = 1$ with (i) All

$\mu_s = \mu_{y_1} = \mu_{y_2} = \dots = \mu_{y_n} = 1$ and (ii) Some $\mu's = 1$.

OR

$$P_{i(Stressed)} = \Phi(\mu_{y_1, \mu_{y_2}, \dots, \mu_{y_n}, d_i) \pm \sigma_s \cdot f(x, \mu, \sigma, \xi) \pm \sigma_{y_1} \cdot f_1(y_1, \mu, \sigma, \pi) \pm \sigma_{y_2} \cdot f_2(y_2, \mu, \sigma, \pi) \pm \dots \pm \sigma_{y_n} \cdot f_n(y_n, \mu, \sigma, \pi)$$

Optimum at:

$$P_{i(Stressed)} = \Phi(d_i) \pm \sigma_s \cdot f(x, \mu, \sigma, \xi) \pm \sigma_{y_1} \cdot f_1(y_1, \mu, \sigma, \pi) \pm \sigma_{y_2} \cdot f_2(y_2, \mu, \sigma, \pi) \pm \dots \pm \sigma_{y_n} \cdot f_n(y_n, \mu, \sigma, \pi)$$

for some $\sigma_{y_1, y_2, \dots, y_n, s} = 1$ with (i) All $\mu_{y_1, y_2, \dots, y_n, s} = 1$ and (ii)

Some $\mu's = 1$. Also, there exists $c, d \in R^+$ such that

$$c < d \text{ then } c < P_{i(Stressed)}^- \leq P_i \leq P_{i(Stressed)}^+ < b.$$

CONCLUSION

The existing Derivatives pricing models overestimates (underestimates) prices especially at the times of economic recessions or recoveries due to their fundamental Normality Assumptions.

Jamilu (2015) has attempted to stress the models by incorporating **ONLY** probability distribution of the underlying stock returns so as to enable them trace the trajectories of the past and future economic and financial crises.

Because of the obvious reasons stated in the material and method of this research paper, hence the sophisticated and holistic stressed extensions.

Generally, by alternating the **TERMS** and **SIGNS (up to 163 and 225)** of all the proposed Jameel's Models, we can observe more **VALUES** that will approximate the existing derivatives models presented in this paper and can be used at the times of Economic and Financial **MELTDOWN** (recoveries and recessions stress periods).

Finally, the sophisticated and holistic extended versions which incorporated the distribution of the underlying stock returns and the strike prices (returns) were found efficiently working and have the ability to traces the trajectories of the potential Black Swan events reference to the derivatives pricing otherwise make them more sophisticated and holistic.

All proposed Jameel's Advanced Stressed Models are expected to tremendously **INCREASES THE PROBABILITIES OF HIGH LOSSES** in the **entire Global Economy and Financial Markets...**

CreditMetrics™ (1997) stated that **“We remind our readers that no amount of sophisticated analytics will replace experience and professional judgment in managing risks. CreditMetrics™ is nothing more than a high-quality tool for the professional risk manager in the financial markets and is not a guarantee of specific results.”**

“If a seatbelt does not provide perfect protection, it still makes sense to wear one, it is better to wear a seatbelt than to not wear one”. It is better off improving **Derivatives Pricing Models** to incorporate **fat – tailed effects** than not.

REFERENCES

- Adegoke Olubummo (1979), Introduction to Real Analysis, Department of Mathematics, University of Ibadan, Girardet Press (W.A) Co; Ibadan.
- Arditti, Fred D. (1996). Derivatives: A Comprehensive Resource for Options, Futures, Interest Rate Swaps, and Mortgage Securities. Boston: Harvard Business School Press. ISBN0-87584560-6.
- Basel Committee on Banking Supervision (2006), International Convergence of Capital Standards, A Revised Framework Comprehensive Version, Bank for International Settlement, CH – 4002 Basel, Switzerland
- Charles Smithson et al (2000), How the Market Values Credit Derivatives, Journal of Lending & Credit Risk Management, March, 2000.
- Jamilu A. Adamu (2013), A Guide to Financial Mathematics and Risk Management for Nigeria, Book Published by Delcon Press, Suleja, Niger, ISBN: 978 – 223 – 529 – 6, First Edition 2013.
- Jamilu A. Adamu (2014), Modern Approach to Financial Risk Management, Book Published by Delcon Press, Suleja, Niger, ISBN: 978-978-942-265-4, First Edition 2014.
- Jamilu A. Adamu (2014), Understanding Financial Risks, Book Published by Delcon Press, Suleja, Niger, ISBN: ISBN: 978-978-942-266-1, First Edition 2014.
- Jamilu Auwalu Adamu (2015), Banking and Economic Advanced Stressed Probability of Default Models, Published in the Asian Journal of Management Sciences, 03(08), 2015, 10-18.
- Jamilu Auwalu Adamu (2015), Global Economic and Financial Crises Advanced Stressed Derivatives Pricing Models, Published in the Asian Journal of Management Sciences, 03(10), 2015, 11-24.
- Jamilu Auwalu Adamu (2015), Global Economic and Financial Crises Best Fitted Fat – Tailed Effects Probability Distributions submitted for Publication in the Asian Journal of Management Sciences.
- Jamilu Auwalu Adamu (2015), Banking and Economic Advanced Stressed Credit Rating Models submitted for Publication the Risk Journals, Credit Risk Journal.
- J P Morgan, The J.P. Morgan Guide to Credit Derivatives, with Contributions from the RiskMetrics Group, Published by Risk.
- J P Morgan, The J.P. Morgan Guide to Credit Derivatives, with Contributions from the RiskMetrics Group, Published by Risk. 25 (2009) 744 – 759
- Nassim N. Taleb et al (2009), Risk Externalities and Too bid to Fail, New York University Polytechnic Institute, 11201, New York, United States.
- Nassim N. Taleb (2012), The Illusion of Thin – Tails under Aggregation, NYU – Poly, January, 2012
- Nassim N. Taleb (2007), Black Swans and the Domains of Statistics, American Statistician, August 2007, Vol. 6I, No. 3
- Professionals Risk Management International Association (PRMIA) Handbook, 2003 and 2010
- Sandica Ana – Maria (2010), Credit Scoring Modelling: A Micro – Macro Approach, Academy of Economic Studies, Doctoral School of Finance DOFIN, 2010

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