

RESEARCH ARTICLE

Received on: 13/07/2015

Accepted on: 10/08/2015

Published on: 20/08/2015

Corresponding Author

Jamilu Auwalu Adamu

National Mathematical Centre, Abuja -
Nigeria

E-mail :

whitehorseconsult@yahoo.com



QR Code for Mobile users

Conflict of Interest: None Declared !

Global Economic and Financial Crises Advanced Stressed Derivatives Pricing Models

Jamilu Auwalu Adamu

National Mathematical Centre, Abuja - Nigeria

ABSTRACT

Black - Scholes Formula has being seriously and widely criticized as the one of the major components that contributed to the 2007 - 2008 financial and economic crises especially in the industry of Financial Derivatives that worth over \$500 trillion before the crises.

The BLACK - SWAN events theory of Nassim N. Taleb has drastically suppresses the over reliance of NORMALITY ASSUMPTIONS almost in all of the Derivatives Pricing Models.

Though, Banks Regulators and Supervisors, Basel II and III dealt comprehensively and extensively about the idea of Stress Testing, meanwhile, the Stress Testing methodologies from sensitivity test, scenario analysis, and historical scenarios to hypothetical scenarios were based on IMPLICIT ASSUMPTIONS. Nevertheless, they contributed immensely in addressing Global Economic and Financial Crises.

Generally, the existing Derivatives Pricing Models (Black - Scholes - Merton Models) did not extensively deal with the behavior of FAT - TAILED EFFECTS of the underlying asset return.

The main aim of this research paper is to consider the existing Derivatives Pricing Models and incorporate FAT - TAILED EFFECTS in them using JAMEEL'S Contractional and Expansional Stress Methods. For the sake of data analysis, data were extracted from yahoo finance (2014 - 1991) of Microsoft Corporation and Chevron Corporation (CVX), Economic Research in the case of U.S. Macroeconomic indicators.

Finally, the new proposed Jameel's Advanced Stressed Derivatives Pricing Models developed were found efficiently working and can capture the low - probability, high - impact events and readily available to predict future Global Economic and Financial Crises associated to Derivatives Pricing given accurate, valid and reasonable models' independent variables.

Keywords: Options, Call, Put, Stress Test, Black - Scholes - Merton, Jameel

Introduction

The Economic and Financial Crisis of the late 2007 - 2008 has brought serious debates among the leading Practitioners and Academics about the incompleteness of the Derivatives Pricing Models. One of the most criticized aspects was the fundamental **NORMALITY** assumption in almost all of the Derivatives Pricing Models. This assumption of course makes the models to either underestimate or overestimate the derivatives prices especially at the times of Economic Recoveries or Recessions.

There were so many attempts by both the markets Practitioners and Academics to improve on the existing derivatives models more especially at the stress periods, unfortunately, up to this date of writing this paper, the models still assumed Normality. Though, **Basel II and III** extensively recommended the idea of Stress Testing which also extremely important concept to address Global Economic and Financial Crises via Derivatives Pricing. The major problem of Stress Testing Methodologies, mostly it deal with **IMPLICIT ASSUMPTION** in stressing the fundamental

macroeconomic indicators, Scenarios, Financial Ratios, PDs, EADs, LGDs and so on.

Recall that **Stress testing** has been adopted as a generic term describing various techniques used by financial firms to gauge their potential vulnerability to exceptional but plausible events. The most common of these techniques involve the determination of the impact on the portfolio of a firm or business unit of a move in a particular market risk factor (**a simple sensitivity test**) or of a simultaneous move in a number of risk factors, reflecting an event which the firm's risk managers believe may occur in the foreseeable future (**scenario analysis**). The scenarios are developed either by drawing on a significant market event experienced in the past (**historical scenarios**) or by thinking through the consequences of a plausible market event which has not yet happened (**hypothetical scenarios**). Other techniques used by some firms to capture their exposure to extreme market events include a **maximum loss approach**, in which risk managers estimate the combination of market moves that would be most damaging to a

portfolio, and **extreme value theory**, which is the statistical theory concerned with the behavior of the “tails” of a distribution of market returns. Thanks to the Idea of Stress Testing and by extension Extreme Value Theory. Nassim N Taleb (2011), emphasizes in most of his papers, the **effects of Low - Probability, High - impact Events** and **incompleteness of prediction models** to accurately capture Financial and Economic crises or chaotic situations in the other field of knowledge.

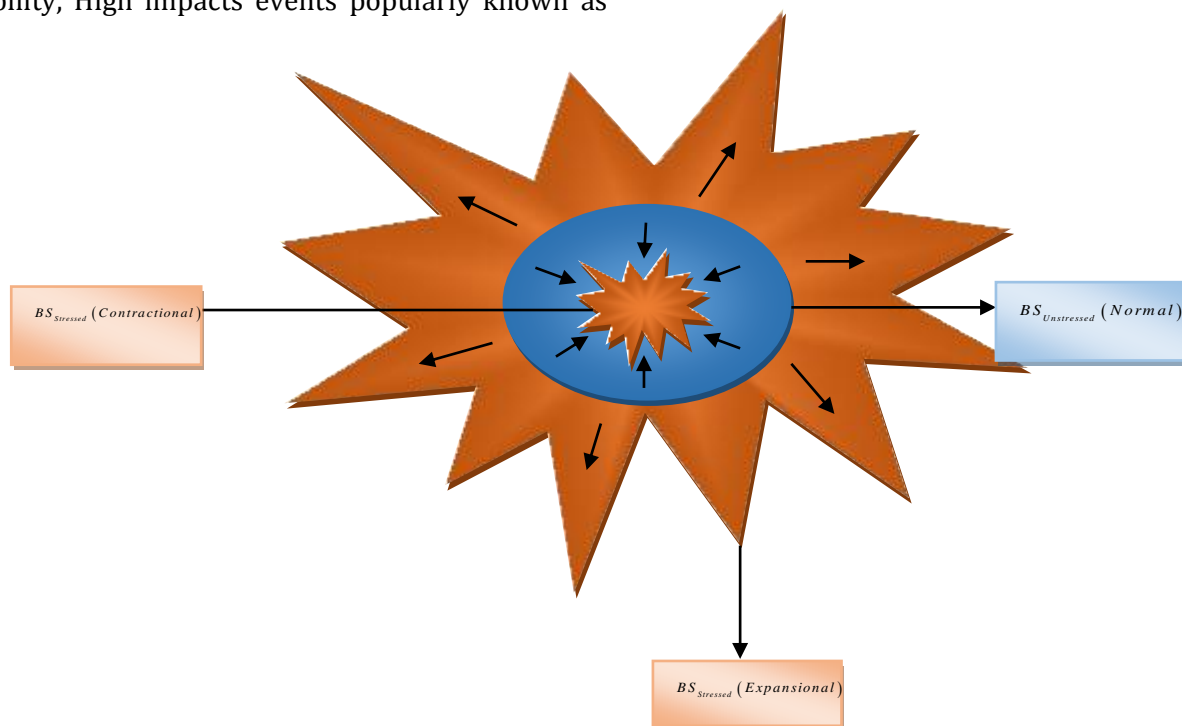
Also, he is the one of the leading actors in the financial markets to propagate the inclusion of “**fat - tail effects**” in the modeling of credit derivatives models. This is of course gave birth to his popular “**Black Swan Idea or Theory**”. He seriously criticizes the **assumption of Normality** in the most of the currently useful Financial and Economic models and in the same vein, the **Black Box assumptions** of probability distributions in the modeling process of financial models.

However, the main aim of this research paper is to consider the Existing Derivatives Pricing Models and apply **JAMEEL’S** Contractual and Expansional Stress Methodology such that they can capture Low - probability, High impacts events popularly known as

BLACK SWAN events so as to predict future Global Economic and Financial Crises given accurate, valid and reasonable models’ independent variables.

MATERIAL AND METHODS

The methodology adopted in this research work is to consider the traditional **Black - Scholes - Merton (1973)** and its Greeks, **Garman - Kohlhagen (1983) Foreign Exchange Options, Black (1976) for Caps, Floors and Swaptions Pricing Models**, and to develop new advanced stressed Derivatives Pricing models that can capture **LOW - PROBABILITY, HIGH - IMPACT** events popularly known as **BLACK SWAN** events by incorporating crises components into them. The **IDEA** was basically on how to **CONTRACTIONALLY** and **EXPANSIONALLY** stress **BLACK - SCHOLES - MERTON** Options Pricing Model using the respectively **Geometric Volatility σ_A** and **Geometric Return μ_A** of the **Arithmetic Means of the Underlying Asset Return and Returns of the explained (Independent) variables** as well as the **Best Fitted Fat - Tailed Effects Probability Distribution of the Underlying Asset Return** as shown below:



I consider the **U.S. Fundamental Macroeconomic Indicators (Independent Variables)**, since **Microsoft Corporation (MSFT)** was listed in the platform of **New York Stock Exchange (NYSE)**. These include: (a) The Monthly Stock Returns of the Microsoft Corporation (b) The U.S. GDP (c) The U.S. Inflation Rate (d) The U.S. Prime Rate (e) The U.S. unemployment

Rate (f) The U.S. USD/GBP Exchange Rate (g) The U.S. House Price (h) The U.S. Oil Price (i) The U.S. Gold Price.

Proposed Jameel’s Models 22:

Recall that the price of **CALL OPTION** is given by:

$$C(S, t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$

The proposed models considering **Black – Scholes – Merton (1973) Model** are given by:

TYPE A (\times): $C(S, t)_{Stressed} = S \left[\Phi(\mu_A d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] - \left[\Phi(\mu_A d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

TYPE A* (*): $C(S, t)_{Stressed} = S \left[\Phi(\mu_A d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] - \left[\Phi(d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

TYPE B (\times): $C(S, t)_{Stressed} = S \left[\Phi(\mu_A d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] - \left[\Phi(\mu_A d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

TYPE B* (*): $C(S, t)_{Stressed} = S \left[\Phi(\mu_A d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] - \left[\Phi(d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

TYPE C: $C(S, t)_{Stressed} = S \left[\Phi(d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] - \left[\Phi(d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

TYPE D: $C(S, t)_{Stressed} = S \left[\Phi(d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] - \left[\Phi(d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

The price of **PUT OPTION** is given by: $P(S, t) = -\Phi(-d_1)S + \Phi(-d_2)K e^{-r(T-t)}$ then

TYPE A: $P(S, t)_{Stressed} = -S \left[\Phi(-\mu_A d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] + \left[\Phi(-\mu_A d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

TYPE A*: $P(S, t)_{Stressed} = -S \left[\Phi(-\mu_A d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] + \left[\Phi(-d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

TYPE B: $P(S, t)_{Stressed} = -S \left[\Phi(-\mu_A d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] + \left[\Phi(-\mu_A d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

TYPE B*: $P(S, t)_{Stressed} = -S \left[\Phi(-\mu_A d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] + \left[\Phi(-d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

TYPE C: $P(S, t)_{Stressed} = -S \left[\Phi(-d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] + \left[\Phi(-d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

TYPE D:

$P(S, t)_{Stressed} = -S \left[\Phi(-d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] + \left[\Phi(-d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi) \right] K e^{-r(T-t)}$

Where, the **Black – Scholes – Merton (1973) model or CALL and PUT** options are given by:

$C(S, t) = \Phi(d_1)S - \Phi(d_2)K e^{-r(T-t)}$ and $P(S, t) = -\Phi(-d_1)S + \Phi(-d_2)K e^{-r(T-t)}$ respectively.

$d_1 = (\ln(S/K) + (r + \sigma^2/2)(T-t)) / \sigma \sqrt{T-t}$ and $d_2 = d_1 - \sigma \sqrt{T-t}$. $\Phi(\cdot)$:

The cumulative distribution function of the standard normal distribution, $T - t$: The time to maturity, S : The spot price of the underlying asset, K : The strike price, r : The risk free rate (annual rate, expressed in terms of continuous compounding), σ : The volatility of returns of the underlying asset; $f(x, \mu_{underlying}, \sigma_{underlying}, \xi)$ is the best fat – tailed effects probability distribution of the underlying asset return; μ_A is the Geometric Return of the Arithmetic Means of the U.S. Macroeconomic Indicators including Microsoft Corporation Stock Return; σ_A is the Geometric Volatility of the Volatilities of the U.S. Macroeconomic Indicators including Microsoft Corporation Stock Return.

Also, note that the Macroeconomic Indicators (Independent Variables) depends on the **OPTION/FUTURES OR STOCK EXCHANGES (COUNTRIES)** in which the **Research Company** has being listed.

Black-Scholes Greeks (Sensitivities) for European Calls and Puts

Greek	Call	Put
Value, V	$S \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2)$	$-S \Phi(-d_1) + K e^{-r(T-t)} \Phi(-d_2)$
Delta, $\Delta = \frac{\partial V}{\partial S}$	$\Phi(d_1)$	$\Phi(d_1) - 1$
Gamma, $\frac{\partial^2 V}{\partial S^2}$	$\frac{\Phi'(d_1)}{\sigma S \sqrt{T-t}}$	$\frac{\Phi'(d_1)}{\sigma S \sqrt{T-t}}$

$$\begin{aligned} \text{Theta, } \frac{\partial V}{\partial t} &= \frac{-\sigma S \Phi'(d_1)}{2\sqrt{T-t}} - rKe^{-r(T-t)}\Phi(d_2) & \frac{-\sigma S \Phi'(-d_1)}{2\sqrt{T-t}} + rKe^{-r(T-t)}\Phi'(-d_2) \\ \text{Vega, } \frac{\partial V}{\partial \sigma} &= S\sqrt{T-t}\Phi'(d_1) & S\sqrt{T-t}\Phi'(d_1) \\ \text{Rho, } \frac{\partial V}{\partial r} &= K(T-t)e^{-r(T-t)}\Phi(d_2) & -K(T-t)e^{-r(T-t)}\Phi(-d_2) \end{aligned}$$

Proposed Jameel's Models (Stressed Greeks) 23:

Greek	Call	Put
Stressed Delta, $\Delta = \frac{\partial V}{\partial S}$	$\Phi(\mu_A d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)$ $\Phi(\mu_A d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)$ $\Phi(d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)$ $\Phi(d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)$	$\Phi(\mu_A d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi) - 1$ $\Phi(\mu_A d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi) - 1$ $\Phi(d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi) - 1$ $\Phi(d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi) - 1$

Proposed working Stressed Greeks for both Call and Put: all the TYPE A to TYPE D of Stressed Delta.

Greek	Call	Put
Stressed Gamma, $\frac{\partial^2 V}{\partial S^2}$	$\frac{\Phi'(\mu_A d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)}{\sigma S \sqrt{T-t}}$ $\frac{\Phi'(\mu_A d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)}{\sigma S \sqrt{T-t}}$ $\frac{\Phi'(d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)}{\sigma S \sqrt{T-t}}$ $\frac{\Phi'(d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)}{\sigma S \sqrt{T-t}}$	$\frac{\Phi'(\mu_A d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)}{\sigma S \sqrt{T-t}}$ $\frac{\Phi'(\mu_A d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)}{\sigma S \sqrt{T-t}}$ $\frac{\Phi'(d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)}{\sigma S \sqrt{T-t}}$ $\frac{\Phi'(d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)}{\sigma S \sqrt{T-t}}$

Proposed working Stressed Theta for both Call and Put: all the TYPE A to TYPE D of the Stressed Gamma.

Greek	Call	Put
Stressed Theta, $\frac{\partial V}{\partial t}$	$-\frac{\sigma S [\Phi'(\mu_A d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]}{2\sqrt{T-t}}$ $-rKe^{-r(T-t)} [\Phi(\mu_A d_2) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]$ $-\frac{\sigma S [\Phi'(\mu_A d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]}{2\sqrt{T-t}}$ $-rKe^{-r(T-t)} [\Phi(\mu_A d_2) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]$ $-\frac{\sigma S [\Phi'(d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]}{2\sqrt{T-t}}$ $-rKe^{-r(T-t)} [\Phi(d_2) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]$ $-\frac{\sigma S [\Phi'(d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]}{2\sqrt{T-t}}$ $-rKe^{-r(T-t)} [\Phi(d_2) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]$	$-\frac{\sigma S [\Phi'(-\mu_A d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]}{2\sqrt{T-t}}$ $+rKe^{-r(T-t)} [\Phi(-\mu_A d_2) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]$ $-\frac{\sigma S [\Phi'(-\mu_A d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]}{2\sqrt{T-t}}$ $+rKe^{-r(T-t)} [\Phi(-\mu_A d_2) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]$ $-\frac{\sigma S [\Phi'(-d_1) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]}{2\sqrt{T-t}}$ $+rKe^{-r(T-t)} [\Phi(-d_2) \pm \sigma_A f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]$ $-\frac{\sigma S [\Phi'(-d_1) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]}{2\sqrt{T-t}}$ $+rKe^{-r(T-t)} [\Phi(-d_2) \pm f(x, \mu_{\text{underlying}}, \sigma_{\text{underlying}}, \xi)]$

Proposed working Stressed Greeks for both Call: TYPE A and TYPE B gives unexpected values and are not recommended. TYPE A* and TYPE B* gives very high values, while TYPE C and TYPE D are recommended.

proposed working stressed Theta for Put: all the types seems working, TYPE A, TYPE B, TYPE A* and TYPE B* largely **UNDERESTIMATE** and **OVERESTIMATE** respectively the original Greeks while TYPE C and TYPE D are extremely recommended.

Proposed working stressed Vega for both Call and Put

Greek	Call	Put
Stressed Vega, $\frac{\partial V}{\partial \sigma}$	$S\sqrt{T-t} [\Phi'(\mu_A d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $S\sqrt{T-t} [\Phi'(\mu_A d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $S\sqrt{T-t} [\Phi'(d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $S\sqrt{T-t} [\Phi'(d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$	$S\sqrt{T-t} [\Phi'(\mu_A d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $S\sqrt{T-t} [\Phi'(\mu_A d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $S\sqrt{T-t} [\Phi'(d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $S\sqrt{T-t} [\Phi'(d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$

Proposed working stressed Rho for both Call and Put: all the TYPE A to TYPE D of the Stressed Vega.

Greek	Call	Put
Stressed Rho, $\frac{\partial V}{\partial r}$	$K(T-t)e^{-r(T-t)} [\Phi(\mu_A d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $K(T-t)e^{-r(T-t)} [\Phi(\mu_A d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $K(T-t)e^{-r(T-t)} [\Phi(d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $K(T-t)e^{-r(T-t)} [\Phi(d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$	$-K(T-t)e^{-r(T-t)} [\Phi(-\mu_A d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $-K(T-t)e^{-r(T-t)} [\Phi(-\mu_A d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $-K(T-t)e^{-r(T-t)} [\Phi(-d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$ $-K(T-t)e^{-r(T-t)} [\Phi(-d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)]$

Proposed working Stressed Greeks for both Call and Put: all the TYPE A to TYPE D of the Stressed Rho.

Proposed Jameel's Models 24:

Recall that the **CALL PRICE OPTIONS** on foreign exchange rates is given by:

$$C(S_0, t) = e^{-r(T-t)} (F\Phi(d_1) - K\Phi(d_2)) \text{ then}$$

The proposed models considering **Garman - Kohlhagen (1983) Foreign Exchange Rates Options Price** are given by:

TYPE A (x): $C(S_0, t)_{Stressed} = e^{-r(T-t)} (F[\Phi(\mu_A d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - K[\Phi(\mu_A d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

TYPE A* (*): $C(S_0, t)_{Stressed} = e^{-r(T-t)} (F[\Phi(\mu_A d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - K[\Phi(d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

TYPE B (x): $C(S_0, t)_{Stressed} = e^{-r(T-t)} (F[\Phi(\mu_A d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - K[\Phi(\mu_A d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

TYPE B* (*): $C(S_0, t)_{Stressed} = e^{-r(T-t)} (F[\Phi(\mu_A d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - K[\Phi(d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

TYPE C: $C(S_0, t)_{Stressed} = e^{-r(T-t)} (F[\Phi(d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - K[\Phi(d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

TYPE D: $C(S_0, t)_{Stressed} = e^{-r(T-t)} (F[\Phi(d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - K[\Phi(d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

The **PUT PRICE OPTIONS** on foreign exchange rates is given by:

$$P(S_0, t) = e^{-r(T-t)} (K\Phi(-d_2) - F\Phi(-d_1)) \text{ then}$$

TYPE A (x): $P(S_0, t)_{Stressed} = e^{-r(T-t)} (K[\Phi(-\mu_A d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - F[\Phi(-\mu_A d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

TYPE A* (*): $P(S_0, t)_{Stressed} = e^{-r(T-t)} (K[\Phi(-\mu_A d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - F[\Phi(-d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

TYPE B (x): $P(S_0, t)_{Stressed} = e^{-r(T-t)} (K[\Phi(-\mu_A d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - F[\Phi(-\mu_A d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

TYPE B* (*): $P(S_0, t)_{Stressed} = e^{-r(T-t)} (K[\Phi(-\mu_A d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - F[\Phi(-d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

TYPE C: $P(S_0, t)_{Stressed} = e^{-r(T-t)} (K[\Phi(-d_2) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - F[\Phi(-d_1) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

TYPE D: $P(S_0, t)_{Stressed} = e^{-r(T-t)} (K[\Phi(-d_2) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)] - F[\Phi(-d_1) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)])$

Proposed Jameel's Models 25:

Recalled that **CAPS Price** is given by:

$$C_{cap}(t) = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times [F(t, T_{i-1}, T_i) \Phi(d_i) - E \times \Phi(d_i - \sigma_i \sqrt{T_{i-1} - t})]$$

The proposed models considering **Black (1976) for CAPS Price** is given by:

TYPE A (x): $C_{cap}(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{aligned} &F(t, T_{i-1}, T_i) (\Phi(\mu_A d_i) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ &- E \times (\Phi(\mu_A (d_i - \sigma_i \sqrt{T_{i-1} - t})) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{aligned} \right]$

TYPE A* (*) :
$$Cap(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} F(t, T_{i-1}, T_i) (\Phi(\mu_A d_i) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -E \times (\Phi(d_i - \sigma_i \sqrt{T_{i-1} - t}) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

TYPE B (x) :
$$Cap(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} F(t, T_{i-1}, T_i) (\Phi(\mu_A d_i) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -E \times (\Phi(\mu_A (d_i - \sigma_i \sqrt{T_{i-1} - t})) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

TYPE B* (*) :
$$Cap(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} F(t, T_{i-1}, T_i) (\Phi(\mu_A d_i) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -E \times (\Phi(d_i - \sigma_i \sqrt{T_{i-1} - t}) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

TYPE C:
$$Cap(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} F(t, T_{i-1}, T_i) (\Phi(d_i) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -E \times (\Phi(d_i - \sigma_i \sqrt{T_{i-1} - t}) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

TYPE D:
$$Cap(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} F(t, T_{i-1}, T_i) (\Phi(d_i) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -E \times (\Phi(d_i - \sigma_i \sqrt{T_{i-1} - t}) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

Proposed Jameel's Models 26:

Recalled that **FLOORS** Price is given by:

$$Floor(t) = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[-F(t, T_{i-1}, T_i) \Phi(-d_i) + E \times \Phi(-d_i + \sigma_i \sqrt{T_{i-1} - t}) \right]$$

The proposed models considering **Black (1976)** for **FLOORS** Price is given by:

TYPE A:
$$Floor(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} -F(t, T_{i-1}, T_i) (\Phi(-\mu_A d_i) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ +E \times (\Phi(\mu_A (-d_i + \sigma_i \sqrt{T_{i-1} - t})) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

TYPE A*:
$$Floor(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} -F(t, T_{i-1}, T_i) (\Phi(-\mu_A d_i) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ +E \times (\Phi(-d_i + \sigma_i \sqrt{T_{i-1} - t}) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

TYPE B:
$$Floor(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} -F(t, T_{i-1}, T_i) (\Phi(-\mu_A d_i) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ +E \times (\Phi(\mu_A (-d_i + \sigma_i \sqrt{T_{i-1} - t})) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

TYPE B*:
$$Floor(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} -F(t, T_{i-1}, T_i) (\Phi(-\mu_A d_i) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ +E \times (\Phi(-d_i + \sigma_i \sqrt{T_{i-1} - t}) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

TYPE C:
$$Floor(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} -F(t, T_{i-1}, T_i) (\Phi(-d_i) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ +E \times (\Phi(-d_i + \sigma_i \sqrt{T_{i-1} - t}) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

TYPE D:
$$Floor(t)_{Stressed} = \sum_{i=1}^n M \times \Delta \times D(t, T_i) \times \left[\begin{array}{l} -F(t, T_{i-1}, T_i) (\Phi(-d_i) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ +E \times (\Phi(-d_i + \sigma_i \sqrt{T_{i-1} - t}) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right]$$

Proposed Jameel's Models 27:

Recalled that the **Payer Swaption** formula is given by:

$$Swaption(t) = \sigma \times M \times \sum_{i=1}^n (F_s(t) \times \Phi(d) - F \times \Phi(d - \sigma_s \sqrt{T_0 - t})) \times D(t, T_i)$$

The proposed models considering **Black (1976)** for **Payer Swaption Prices** are given by:

TYPE A (x) :
$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left[\begin{array}{l} F_s(t) \times (\Phi(\mu_A d) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -F \times (\Phi(\mu_A (d - \sigma_s \sqrt{T_0 - t})) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right] \times D(t, T_i)$$

TYPE A* (*) :
$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left[\begin{array}{l} F_s(t) \times (\Phi(\mu_A d) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -F \times (\Phi(d - \sigma_s \sqrt{T_0 - t}) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right] \times D(t, T_i)$$

TYPE B (x) :
$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left[\begin{array}{l} F_s(t) \times (\Phi(\mu_A d) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -F \times (\Phi(\mu_A (d - \sigma_s \sqrt{T_0 - t})) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right] \times D(t, T_i)$$

TYPE B* (*) :
$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left[\begin{array}{l} F_s(t) \times (\Phi(\mu_A d) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -F \times (\Phi(d - \sigma_s \sqrt{T_0 - t}) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right] \times D(t, T_i)$$

TYPE C:
$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left[\begin{array}{l} F_s(t) \times (\Phi(d) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -F \times (\Phi(d - \sigma_s \sqrt{T_0 - t}) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right] \times D(t, T_i)$$

TYPE D:
$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left[\begin{array}{l} F_s(t) \times (\Phi(d) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ -F \times (\Phi(d - \sigma_s \sqrt{T_0 - t}) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{array} \right] \times D(t, T_i)$$

The pricing formula for the **Receiver Swaptions** is given by:

Swaption(t) = $\sigma \times M \times \sum_{i=1}^n (-F_s(t) \times \Phi(-d) + F \times \Phi(-d + \sigma_s \sqrt{T_0 - t})) \times D(t, T_i)$ then

TYPE A:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left(\begin{aligned} & -F_s(t) \times (\Phi(-\mu_A d) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ & + F (\Phi(\mu_A (-d + \sigma_s \sqrt{T_0 - t})) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{aligned} \right) \times D(t, T_i)$$

TYPE A*:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left(\begin{aligned} & -F_s(t) \times (\Phi(-\mu_A d) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ & + F (\Phi(-d + \sigma_s \sqrt{T_0 - t}) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{aligned} \right) \times D(t, T_i)$$

TYPE B:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left(\begin{aligned} & -F_s(t) \times (\Phi(-\mu_A d) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ & + F (\Phi(\mu_A (-d + \sigma_s \sqrt{T_0 - t})) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{aligned} \right) \times D(t, T_i)$$

TYPE B*:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left(\begin{aligned} & -F_s(t) \times (\Phi(-\mu_A d) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ & + F (\Phi(-d + \sigma_s \sqrt{T_0 - t}) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{aligned} \right) \times D(t, T_i)$$

TYPE C:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left(\begin{aligned} & -F_s(t) \times (\Phi(-d) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ & + F (\Phi(-d + \sigma_s \sqrt{T_0 - t}) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{aligned} \right) \times D(t, T_i)$$

TYPE D:

$$Swaption(t)_{Stressed} = \sigma \times M \times \sum_{i=1}^n \left(\begin{aligned} & -F_s(t) \times (\Phi(-d) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \\ & + F (\Phi(-d + \sigma_s \sqrt{T_0 - t}) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)) \end{aligned} \right) \times D(t, T_i)$$

RESULT AND DISCUSSION

Having presented the proposed Global Economic and Financial Crises Advanced Stressed Derivatives Pricing Models and that the values of $\mu_{company}$, $\sigma_{company}$, μ_A and σ_A of each of the research company are computed then we are now ready to implement and present the Results of the proposed Jameel Models from 22 to 26, discuss them and forward a new research Theorems derived from them.

Example 1: Consider an example of **Microsoft Corporation (MSFT)** option with a term of six months (0.5 years). The current stock price of **Microsoft Corporation (MSFT)** is \$48.14 and the strike of the option is \$49.39. The risk-free rate is 3.92% p.a. The volatility of the stock is 2.2041976% p.a. What is the value of the options using: (1) Black-Scholes - Merton Model; and (2) Jameel's Economic and Financial Crises Advanced Stressed Derivatives Models reference to Black-Scholes - Merton Model?

Using the data of **Microsoft Corporation (MSFT)** extracted from yahoo finance from 2014 - 1991, we obtained:

$$f(x, \mu_{underlying}, \sigma_{underlying}, \xi) = 0.00000000123523 \text{ (Log - Logistic (3P))}, \mu_A = 0.031886784, \sigma_A = 0.117906073,$$

$$K = \$49.39, S = \$48.14, \sigma = 0.022041976, r = 0.0392, T = 0.5 \text{ and } t = 0. \text{ Recall that}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t} \quad \text{then} \quad d_1 = \frac{\ln\left(\frac{48.14}{49.39}\right) + \left(0.0392 + \frac{0.022041976^2}{2}\right)(0.5-0)}{0.022041976\sqrt{0.5-0}} = -0.18969$$

$$d_2 = -0.18969 - 0.022041976\sqrt{0.5} = -0.20528$$

Therefore, $d_1 = -0.18969$ and $d_2 = -0.20528$. Using the **Microsoft EXCEL**, consider the following tables: **Table**

1: Black - Scholes -Merton and Jameel Advanced Stressed Call Option Prices:

TYPE A+	TYPE A*+	TYPE B+	TYPE B*+	TYPE C+	TYPE D+
0.1353827	3.6767467	-0.135382736	3.6767467	0.1716279	0.1716279
		BLACK - SCHOLES - MERTON			
		0.171627934			
TYPE A-	TYPE A*-	TYPE B-	TYPE B*-	TYPE C-	TYPE D-
0.1353827	3.6767467	-0.135382735	3.6767467	0.1716279	0.1716279

Table 2: Black - Scholes -Merton and Jameel Advanced Stressed Put Option Prices:

TYPE A+	TYPE A*+	TYPE B+	TYPE B*+	TYPE C+	TYPE D+
0.1559984	3.9681278	0.155998418	3.9681278	0.4630091	0.4630091
		BLACK - SCHOLES - MERTON			
		0.463009087			
TYPE A-	TYPE A*-	TYPE B-	TYPE B*-	TYPE C-	TYPE D-
0.1559984	3.9681278	0.155998417	3.9681278	0.4630091	0.4630091

Example 2: Assume the interest on loan is at 6.27% *p.a* compounded quarterly. Suppose that this contract is a caplet with notional value of \$15,000,000 designed to cap the interest rate for a period of three-month starting six months from now. Assume that the forward rate for three-month period starting on six months is 5.08% *p.a.*, compounding quarterly with the volatility of the rate equals 20% *p.a* . What are the prices of **Caplet** and **Flolet** using: (i) **Black Models (1976)** (ii) **Jameel’s Economic and Financial Crises Advanced Stressed Derivatives Models** reference to Black (1976) Model?

$$\sum M = \$15,000,000, \Delta = T_i - T_{i-1} = \frac{1}{4} = 0.25, F = 5.08\% = 0.0508, E = 6.27\% = 0.0627, \sigma = 20\% = 0.20,$$

$$\sigma_A = 0.111414539, \mu_A = 0.030383975, f(x, \mu_{Company}, \sigma_{Company}, \xi) = 0.000073492, \text{ then}$$

$$D = \frac{1}{(1 + \Delta \times F)} = \frac{1}{1 + (0.25 \times 0.0508)} = \frac{1}{1.0127} = 0.99$$

$$d = \frac{\ln\left(\frac{F}{E}\right) + \frac{1}{2} \times \sigma^2 (T - t) - \ln\left(\frac{0.0508}{0.0627}\right) + (0.5 \times (0.20)^2 \times 0.5)}{\sigma \sqrt{T - t}} = \frac{0.01 - 0.2105}{0.20 \times \sqrt{0.5}} = \frac{0.01 - 0.2105}{0.1414} = -1.4178$$

. Using **Microsoft EXCEL**, consider the

following tables:

Table 3: Merton and Jameel Advanced Stressed Caps Prices:

TYPE A+	TYPE A*+	TYPE B+	TYPE B*+	TYPE C+	TYPE D+
20932.104	77213.465	-20934.98954	77210.58	890.02361	887.13856
		MERTON CAPS PRICE			
		890.3853496			
TYPE A-	TYPE A*-	TYPE B-	TYPE B*-	TYPE C-	TYPE D-
20931.381	77214.189	-20928.49597	77217.074	890.74709	893.63213

Table 4: Merton and Jameel Advanced Stressed Floors Prices:

TYPE A+	TYPE A*+	TYPE B+	TYPE B*+	TYPE C+	TYPE D+
23247.369	121392.94	23250.25403	121395.82	45069.497	45072.382
		MERTON CAPS PRICE			
		45069.13535			
TYPE A-	TYPE A*-	TYPE B-	TYPE B*-	TYPE C-	TYPE D-
23246.646	127872.52	23243.76046	121389.33	45068.774	45065.889

From **Example 1 reference to table 1 above** , Jameel's Advanced Stressed Call Option Prices of TYPE A and TYPE B **CAN NOT** be used to price Call Option (negative values for prices), while TYPE A* and TYPE B* extremely **OVERESTIMATE** the original Black – Scholes (1973) approximation, nevertheless could be useful in some markets circumstances.

However, TYPE C and TYPE D are extremely recommended to price Call Option more especially at the times of Economic Recoveries and Recessions since they provide very close approximations to the original Black – Scholes approximation.

From **Example 1 reference to table 2 above**, Jameel's Advanced Stressed Put Option are all positive prices and can be used to price Put option, however TYPE A* and TYPE B* extremely **OVERESTIMATE** the original Black – Scholes approximation, TYPE A and TYPE B prices are almost $\frac{1}{3}$ Price of the original Black – Scholes

approximation, nevertheless could be useful in some other markets circumstances.

TYPE C and TYPE D are extremely recommended to price Put Option more especially at the times of Economic Recoveries and Recessions, since they provide very **CLOSE** approximations to that of original Black – Scholes Put Option Price.

Proposed Jameel's Theorem 6:

Let $B(S, t)_{Normal}$ be a Black – Scholes – Merton Options Pricing Models (Call or Put), let $a, b \in R^+$ to replace $a, b \in \square^+$ such that $0 < b < a$ then there exists some Jameel's Advanced Stressed Options Pricing Models $J^1(S, t)_{Stressed}$ and $J^2(S, t)_{Stressed}$ such that $b \leq J^1(S, t)_{Stressed} \leq B(S, t)_{Normal} \leq J^2(S, t)_{Stressed} \leq a$.

Jameel's Advanced Stressed forex Call Option Prices of TYPE A and TYPE B **CAN NOT** be used to price Call Option (negative values for prices), while TYPE A* and TYPE B* extremely **OVERESTIMATE** the original **Garman - Kohlhagen (1983) Options on Foreign Exchange Rates Prices** approximation, nevertheless could be useful in some markets circumstances.

However, TYPE C and TYPE D are extremely recommended to price Call Option more especially at the times of Economic Recoveries and Recessions since they provide very close approximations to the original **Garman - Kohlhagen (1983) Options on Foreign Exchange Rates Price** approximation.

While, Jameel's Advanced Stressed Put Option are all positive prices and can be used to price Put option, however TYPE A* and TYPE B* extremely **OVERESTIMATE** the original **Garman - Kohlhagen (1983) Options on Foreign Exchange Rates Price**, TYPE A and TYPE B prices are almost $\frac{1}{n}$, $n \in N$ Price of the **Garman - Kohlhagen (1983)**

Options on Foreign Exchange Rates Price approximation, nevertheless could be useful in some other markets circumstances.

TYPE C and TYPE D are extremely recommended to price Put Option more especially at the times of Economic Recoveries and Recessions, since they provide very **CLOSE** approximations to that of original **Garman - Kohlhagen (1983) Options on Foreign Exchange Rates Price** Put Option Price.

Example 2 reference to table 3 above, Jameel's Advanced Stressed Caps Prices of TYPE A and TYPE B **CAN NOT** be used to price Caps (negative values for prices), while TYPE A* and TYPE B* extremely **OVERESTIMATE** the original Black (1976) approximation, nevertheless could be useful in some markets circumstances.

However, TYPE C and TYPE D are extremely recommended to price Caps more especially at the times of Economic Recoveries and Recessions since they provide very close approximations to the original Black (1976) approximation.

Example 2 reference to table 4 above, Jameel's Advanced Stressed Floors are all positive prices and can be used to price Floors, however TYPE A* and TYPE B* extremely **OVERESTIMATE** the original Black (1976) approximation, TYPE A and TYPE B prices are almost $\frac{1}{2}$ Price of the original Black (1976) approximation,

nevertheless could be useful in some other markets circumstances.

TYPE C and TYPE D are extremely recommended to price Floors more especially at the times of Economic Recoveries and Recessions, since they provide very **CLOSE** approximations to that of original Black (1976) Floors Price.

Proposed Jameel's Theorem 7:

Let $M(F, t)_{Normal}$ be a Black Caps (Floors) Pricing Models, let $m, n \in R^+$ such that $0 < n < m$ then there exists some Jameel's Advanced Stressed Caps (Floors) Pricing Models $J^3(F, t)_{Stressed}$ and $J^4(F, t)_{Stressed}$ such that $n \leq J^3(F, t)_{Stressed} \leq M(F, t)_{Normal} \leq J^4(F, t)_{Stressed} \leq m$.

Jameel's Advanced Stressed Payer Swaptions Prices of TYPE A and TYPE B **CAN NOT** be used to price Payer Swaptions (negative values for prices), while TYPE A* and TYPE B* extremely **OVERESTIMATE** the original Black (1976) approximation, nevertheless could be useful in some markets circumstances.

However, TYPE C and TYPE D are extremely recommended to price Payer Swaptions more especially at the times of Economic Recoveries and Recessions since they provide very close approximations to the original Black (1976) approximation.

Jameel's Advanced Stressed Receiver Swaptions are all positive prices and can be used to price Receiver Swaptions, however TYPE A* and TYPE B* extremely **OVERESTIMATE** the original **Black (1976)**, TYPE A and TYPE B prices

are almost $\frac{1}{n}$, $n \in N$ Price of the **Black (1976)** approximation, nevertheless could be useful in some other markets circumstances.

TYPE C and TYPE D are extremely recommended to price Receiver Swaptions more especially at the times of Economic Recoveries and Recessions, since they provide very **CLOSE** approximations to that of original **Black (1976)** Receiver Swaptions Price.

Proposed Jameel's Theorem 8:

Let $N(F, t)_{Normal}$ be a Black (1976) Payer Swaptions (Receiver Swaptions) Pricing Models, let $c, d \in R^+$ such that $0 < c < d$ then there exists some Jameel's Advanced Stressed Payer Swaptions (Buyer Swaptions) Pricing Models $J^5(F, t)_{Stressed}$ and $J^6(F, t)_{Stressed}$ such that $c \leq J^5(F, t)_{Stressed} \leq N(F, t)_{Normal} \leq J^6(F, t)_{Stressed} \leq d$.

Proposed Theorem 9 (Jameel Maths):

Let M_{Normal} be any stochastic financial or economic model (in Modern Portfolio Theory, VaR, Bonds, Stocks, ETFs, Futures, Exotic Options, Credit Derivatives and so on). Let $P_i (i = 1, \dots, n)$ be the independent probabilities of M_{Normal} ; let $k, l \in R^+$; $\mu_{underlying}$ be the Mean of the underlying Asset Return; $\sigma_{underlying}$ be the volatility of the underlying Asset Return; μ_A be the Geometric Mean of the Arithmetic Means of the Macroeconomic indicators including research company underlying Asset Return; σ_A be the Geometric Volatility of the Volatilities of the Macroeconomic indicators including research company underlying Asset Return; $f(x, \mu_{underlying}, \sigma_{underlying}, \xi)$ be any **BEST FITTED FAT - TAILED EFFECTS** probability distribution of the underlying Asset Return then M_{Normal} can be **Contractionally** or **Expansionally STRESSED** by **REPLACING** P_i 's ($i = 1, \dots, n$) accordingly with the following **JAMEEL'S** substitutions:

(i) $P_i \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi); i = 1, \dots, n$

(ii) $P_i \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi); i = 1, \dots, n$

(iii) If e^{y_i} such that y_i 's $\in R^+$ appears in the denominator of P_i such that $P_i = \frac{1}{1 + e^{y_i}}; (i = 1, \dots, n)$ then e^{y_i} 's

can be substituted by:

(a) $e^{\mu_A \cdot y_i} \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi); i = 1, \dots, n$

(b) $e^{\mu_A \cdot y_i} \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi); i = 1, \dots, n$

(c) $e^{y_i} \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi); i = 1, \dots, n$

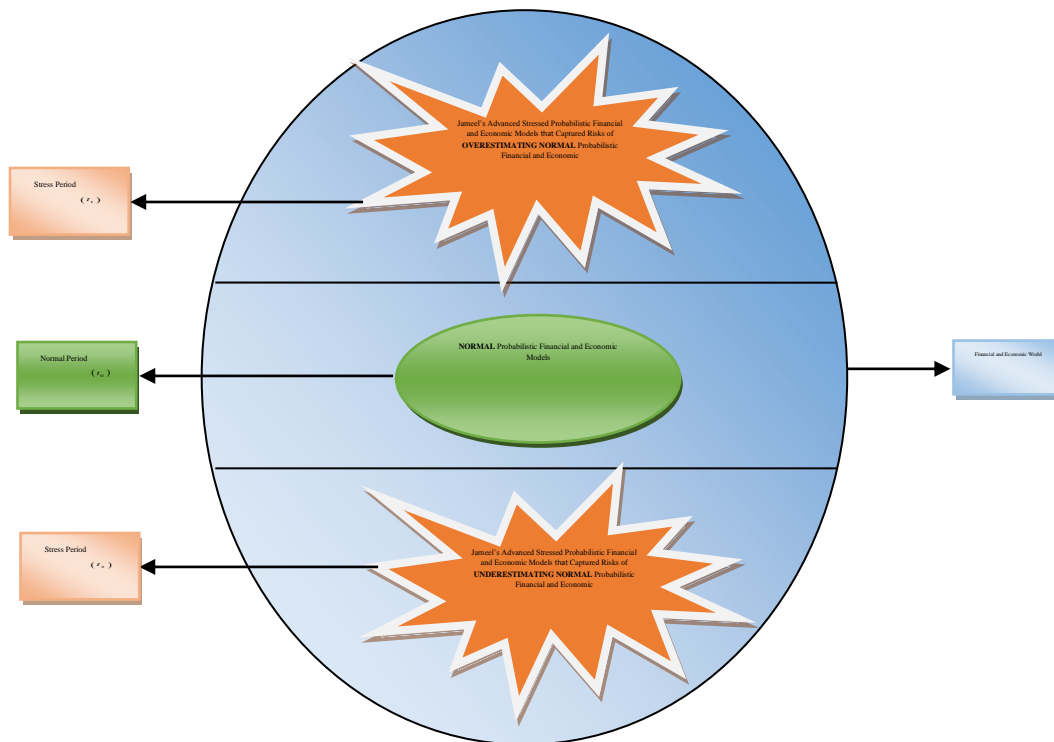
(d) $e^{y_i} \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi); i = 1, \dots, n$

(iv) If $P_i = \Phi(d_i)$; ($i = 1, \dots, n$) then P_i 's can be substituted by:

- (a) $\Phi(\mu_A \cdot d_i) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)$; $i = 1, \dots, n$
- (b) $\Phi(\mu_A \cdot d_i) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)$; $i = 1, \dots, n$
- (c) $\Phi(d_i) \pm \sigma_A f(x, \mu_{underlying}, \sigma_{underlying}, \xi)$; $i = 1, \dots, n$
- (d) $\Phi(d_i) \pm f(x, \mu_{underlying}, \sigma_{underlying}, \xi)$; $i = 1, \dots, n$

Also, there exists some **JAMEEL'S** probability functions J_i^1 and J_i^2 and **JAMEEL'S** Advanced Stressed Global Economic and Financial Stochastic Models $J(M_1)_{Stressed}$ and $J(M_2)_{Stressed}$ such that

$$0 \leq J_i^1 \leq P_i \leq J_i^2 \leq 1; i = 1, \dots, n \text{ and } l \leq J(M_1)_{Stressed} \leq M_{Normal} \leq J(M_2)_{Stressed} \leq k.$$



Jameel's Maths Theorem can be applied in order to **STRESS** probability models or models that depend on probabilities in other fields of **SCIENCES, ARTS, SOCIAL AND MANAGEMENT SCIENCES, MEDICINE AND ENGINEERING**. However, in those situations μ_A and σ_A depends on the fundamental independent variables that measured the field's stability (as in the case of macroeconomic indicators), whereas $f(x, \mu_{underlying}, \sigma_{underlying}, \xi)$ depends on the research item underlying factor. **JAMEEL'S MATHS** theorem is in line with what the **BLACK SWAN** theory is advocating.

Proposed Theorem (Jameel's SAYF's Envy):

(i) If $f(x, \mu_{company}, \sigma_{company}, \xi) \equiv 0$ in (a) to (d) of (iii) of **Jameel's Maths Theorem** above, then

- (a) $P_i = \frac{1}{1 + \exp(\mu_A y_i)}$; $i = 1, 2, \dots, n$, for TYPE A and TYPE B
- (b) $P_i = \frac{1}{1 + \exp(y_i)}$; $i = 1, 2, \dots, n$, for TYPE C and TYPE D

Thus, (a) of (i) above are probabilities of Default approximations without **fat – tailed effects** in the neighborhood of P_i 's ($i = 1, 2, \dots, n$) **NORMAL CLASS**.

(ii) If $f(x, \mu_{company}, \sigma_{company}, \xi) \equiv 0$ in (a) to (d) of (iv) of **Jameel's Maths Theorem** above, then

(a) $P_i = \Phi (\mu_A d_i)$; $i = 1, 2, \dots, n$, for TYPE A and TYPE B

(b) $P_i = \Phi (d_i)$; $i = 1, 2, \dots, n$, for TYPE C and TYPE D

Thus, (a) of (ii) above are probabilities of Default approximations without **fat - tailed effects** in the neighborhood of P_i 's ($i = 1, 2, \dots, n$) **NORMAL CLASS**.

(iii) If $f (t, \mu_{company}, \sigma_{company}, \xi) \equiv 0$ for Instantaneous Probability of Survival Model, then

(a) $PS = \exp \left\{ - \int_0^T \mu_A \lambda (t) dt \right\}$; for TYPE A and TYPE B

(b) $PS = \exp \left\{ - \int_0^T \lambda (t) dt \right\}$; for TYPE C and TYPE D

Thus, (a) of (iii) above are probabilities of Survival approximations without **fat - tailed effects** in the neighborhood of PS 's **NORMAL CLASS**.

Proposed Theorem (Jameel's Charles Confession):

Let $A \subset R^+$ and $B \subset R^+$ such that $M (x)_{Normal} : A \rightarrow B$ be any Deterministic Financial or Economic Model, let μ_A be the Geometric Return of the Arithmetic Means of the Company Based Country's Macroeconomic Indicators including Company underlying Stock Return and σ_A be the Geometric Volatility of the Volatilities of the Company Based Country's Macroeconomic Indicators including Company underlying Stock Return. Let u be the uncertainty component of $M (x)_{Normal}$ such that du is infinitesimal positive, let $\varepsilon > 0$ be infinitesimal positive such that

$$|M (x)_{Normal}| - \varepsilon \leq |M (x)_{Normal}| \leq |M (x)_{Normal}| + \varepsilon ; \forall x \in A$$

Where, $\varepsilon = \mu_A d (M (x)_{Normal}) + \sigma_A du$, since $\frac{d (M (x)_{Normal})}{dx} = M '(x)_{Normal}$

$\Rightarrow d (M (x)_{Normal}) = M '(x)_{Normal} \cdot dx$, thus $\varepsilon = \mu_A (M '(x)_{Normal} \cdot dx) + \sigma_A du$ then we can **CATASTROPHICALLY STRESSED** $M (x)_{Normal}$ as follows:

(i) $M (x)_{Normal}$ is said to be **EXPANSIONALLY STRESSED** if $|M (x)_{Normal}| \leq |M (x)_{Normal}| + \varepsilon ; \forall x \in A$ satisfying the following **JAMEEL'S** substitutions:

TYPE A: $|M (x)_{Normal}| \leq |M (x)_{Normal}| + [\mu_A |M '(x)_{Normal} \cdot dx| \pm \sigma_A du] = M (x)_{Stressed} ; \forall x \in A$

TYPE B: $|M (x)_{Normal}| \leq |M (x)_{Normal}| + [\mu_A |M '(x)_{Normal} \cdot dx| \pm du] = M (x)_{Stressed} ; \forall x \in A$

TYPE C: $|M (x)_{Normal}| \leq |M (x)_{Normal}| + [|M '(x)_{Normal} \cdot dx| \pm \sigma_A du] = M (x)_{Stressed} ; \forall x \in A$

TYPE D: $|M (x)_{Normal}| \leq |M (x)_{Normal}| + [|M '(x)_{Normal} \cdot dx| \pm du] = M (x)_{Stressed} ; \forall x \in A$

(ii) $M (x)_{Normal}$ is said to be **CONTRACTIONALLY STRESSED** if $|M (x)_{Normal}| - \varepsilon \leq |M (x)_{Normal}| ; \forall x \in A$ satisfying the following **JAMEEL'S** substitutions:

TYPE A: $M (x)_{Stressed} = |M (x)_{Normal}| - [\mu_A |M '(x)_{Normal} \cdot dx| \pm \sigma_A du] \leq |M (x)_{Normal}| ; \forall x \in A$

TYPE B: $M (x)_{Stressed} = |M (x)_{Normal}| - [\mu_A |M '(x)_{Normal} \cdot dx| \pm du] \leq |M (x)_{Normal}| ; \forall x \in A$

TYPE C: $M (x)_{Stressed} = |M (x)_{Normal}| - [|M '(x)_{Normal} \cdot dx| \pm \sigma_A du] \leq |M (x)_{Normal}| ; \forall x \in A$

TYPE D: $M (x)_{Stressed} = |M (x)_{Normal}| - [|M '(x)_{Normal} \cdot dx| \pm du] \leq |M (x)_{Normal}| ; \forall x \in A$

Thus,

TYPE A:

$$M (x)_{Contractionally Stressed} = |M (x)_{Normal}| - [\mu_A |M '(x)_{Normal} \cdot dx| \pm \sigma_A du] \leq |M (x)_{Normal}| \leq |M (x)_{Normal}| + [\mu_A |M '(x)_{Normal} \cdot dx| \pm \sigma_A du] = M (x)_{Expansionally Stressed} ; \forall x \in A$$

TYPE B:

$$M (x)_{Contractionally Stressed} = |M (x)_{Normal}| - [\mu_A |M '(x)_{Normal} \cdot dx| \pm du] \leq |M (x)_{Normal}| \leq |M (x)_{Normal}| + [\mu_A |M '(x)_{Normal} \cdot dx| \pm du] = M (x)_{Expansionally Stressed} ; \forall x \in A$$

TYPE C:

$$M(x)_{\text{Contractually Stressed}} = \left| M(x)_{\text{Normal}} \right| - \left[\left| M'(x)_{\text{Normal}} \cdot dx \right| \pm \sigma_A du \right] \leq \left| M(x)_{\text{Normal}} \right| \leq \left| M(x)_{\text{Normal}} \right| + \left[\left| M'(x)_{\text{Normal}} \cdot dx \right| \pm \sigma_A du \right] = M(x)_{\text{Expansionally Stressed}} ; \forall x \in A$$

TYPE D:

$$M(x)_{\text{Contractually Stressed}} = \left| M(x)_{\text{Normal}} \right| - \left[\left| M'(x)_{\text{Normal}} \cdot dx \right| \pm du \right] \leq \left| M(x)_{\text{Normal}} \right| \leq \left| M(x)_{\text{Normal}} \right| + \left[\left| M'(x)_{\text{Normal}} \cdot dx \right| \pm du \right] = M(x)_{\text{Expansionally Stressed}} ; \forall x \in A$$

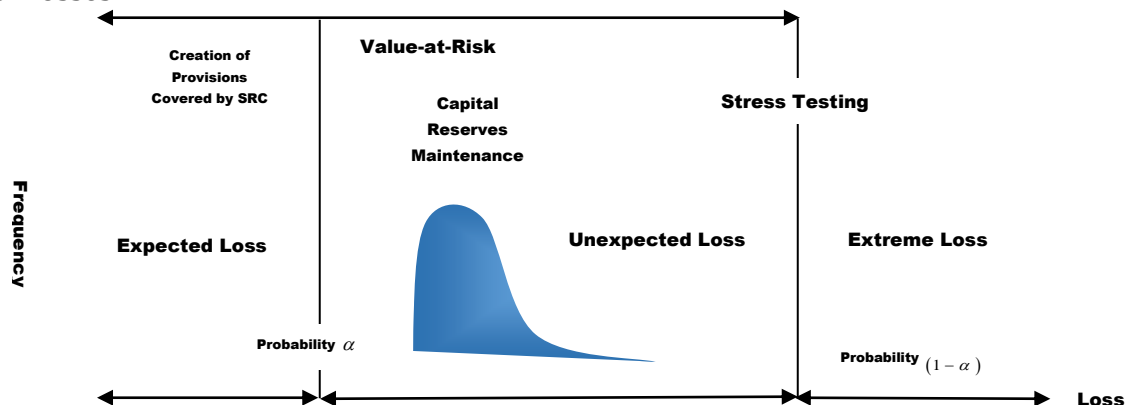
Jameel’s Charles Confession Theorem categorically stated that **ALL** deterministic Financial and Economic Models in the form $M(x)_{\text{Normal}}$ working at the **NORMAL** markets conditions can be **EXPLICITLY STRESSED** to work at the times of Economic Recoveries and Recessions or Financial and Economic Crises.

The idea of Jameel’s Charles Confession Theorem can be directly applied in other fields of **SCIENCES, ARTS, SOCIAL AND MANAGEMENT SCIENCES, MEDICINE AND ENGINEERING** to capture chaotic situations or to increase the X-ray power of the models’ sensitivities to change.

Proposed Theorem (Bash – Eves – Kali’s Sacrifice):

Consider the following diagram:

Basel II Distribution of Losses

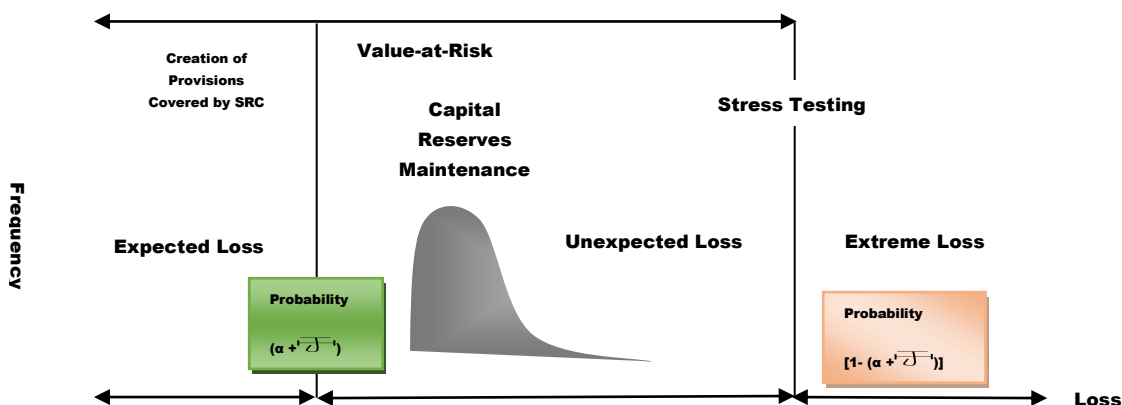


Let **DB** be the distribution of losses under **BASEL II accord** described in the above diagram. Let $J(M)$'s be all Jameel’s Advanced Stressed Stochastic and Deterministic Financial and Economic models obtained as the result of Jameel’s **CONTRACTIONAL** and **EXPANSIONAL** stress methods, let α and $(1-\alpha)$ be **Normal Markets Confidence and Significant levels** respectively, let ${}^i\mathcal{J}^{-1} \in R^+$ be an infinitesimal positive constant strictly less than zero then $J(M)$'s Advanced Stressed Stochastic and Deterministic Financial and Economic models have increases Markets **CONFIDENCE LEVEL** by $(+{}^i\mathcal{J}^{-1})$ and decreases Markets **SIGNIFICANT LEVEL** by $(-{}^i\mathcal{J}^{-1})$, meaning now, Markets Confidence and Significant levels have consequently became $(\alpha + {}^i\mathcal{J}^{-1})$ and $[1 - (\alpha + {}^i\mathcal{J}^{-1})]$ respectively. Where, ${}^i\mathcal{J}^{-1}$ is called **JAMEEL’S CONSTANT**.

Interpretation and Conclusion: Bash – Eves – Kali’s Sacrifice Theorem has increases **MARKETS CONFIDENCE** dramatically and reduces **MARKETS RISKS** drastically.

Proposed Eve’s Transition Diagram:

New Proposed Basel II Distribution of Losses



CONCLUSION

Black – Scholes Formula has being seriously and widely criticized as the one of the major components that contributed to the 2007 – 2008 financial crises especially in the industry of financial derivatives that worth over \$500 trillion before the crisis. No, the use of the formula has being **ABUSED**, because as the time passed, the level of confidence grew exponentially, many bankers, traders and academics forgot that the model had limitations vis-à-vis the backbone **NORMALITY ASSUMPTIONS**.

The Black Monday of 19 October, 1987 is an event that world's stock market lost more than 20% of their value within a few hours. This is an **EXTREME** event that is virtually impossible to happen under the models' assumptions (especially the Normality assumption).

Nassim N. Taleb calls this low – probability, high impact or extreme event popularly known as **BLACK SWAN** event. The Black – Scholes Normality assumption **IGNORED** potential Black Swans otherwise the model performed wonderfully very well (Thanks to the Black – Scholes – Merton generous invention).

However, in this research work, I proposed **NEW** advanced stressed Derivatives pricing models that incorporated **FAT – TAILED EFFECTS** (low – probability, high impact) reference to the popular Black – Scholes – Merton (1973), Merton (1976) and Garman – Kohlhagen (1983) in an attempt to capture the popular Black Swan events **otherwise make them more sophisticated**.

These proposed advanced stressed models were found efficiently working using the numerical examples above and are readily available to capture future Black Swan events (**otherwise make them more sophisticated**) and avoid future Derivatives global economic and financial crises given accurate, valid and reasonable models' independent variables.

CreditMetrics™ (1997) stated that **“We remind our readers that no amount of sophisticated analytics will replace experience and professional judgment in managing risks.** CreditMetrics™ is nothing more than a high-quality tool for the professional risk

manager in the financial markets and is not a guarantee of specific results.”

“If a seatbelt does not provide perfect protection, it still makes sense to wear one, it is better to wear a seatbelt than to not wear one”. It is better off improving **Derivatives Pricing Models** to incorporate **fat – tailed effects** than not.

REFERENCES

Basel Committee on Banking Supervision (2006), International Convergence of Capital Standards, A Revised Framework Comprehensive Version, Bank for International Settlement, CH – 4002 Basel, Switzerland
Charles Smithson et al (2000), How the Market Values Credit Derivatives, Journal of Lending & Credit Risk Management, March, 2000.

Jamilu A. Adamu (2013), A Guide to Financial Mathematics and Risk Management for Nigeria, Book Published by Delcon Press, Suleja, Niger, ISBN: 978 – 223 – 529 – 6, First Edition 2013.

Jamilu A. Adamu (2014), Modern Approach to Financial Risk Management, Book Published by Delcon Press, Suleja, Niger, ISBN: 978-978-942-265-4, First Edition 2014.

Jamilu A. Adamu (2014), Understanding Financial Risks, Book Published by Delcon Press, Suleja, Niger, ISBN: ISBN: 978-978-942-266-1, First Edition 2014.

Jamilu Auwalu Adamu (2015), Banking and Economic Advanced Stressed Probability of Default Models, Published in the Asian Journal of Management Sciences, AJMS/2015/ May – June – 303 – 798 – 2 – RV.
J P Morgan, The J.P. Morgan Guide to Credit Derivatives, with Contributions from the RiskMetrics Group, Published by Risk.

J P Morgan, The J.P. Morgan Guide to Credit Derivatives, with Contributions from the RiskMetrics Group, Published by Risk. 25 (2009) 744 – 759
Professionals Risk Management International Association (PRMIA) Handbook, 2003 and 2010

Sandica Ana – Maria (2010), Credit Scoring Modelling: A Micro – Macro Approach, Academy of Economic Studies, Doctoral School of Finance DOFIN, 2010

Cite this article as:

Jamilu Auwalu Adamu. Global Economic and Financial Crises Advanced Stressed Derivatives Pricing Models. Asian Journal of Management Sciences, 03(10), 2015, 11-24.