RESEARCH ARTICLE

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Banking and Economic Advanced Stressed Probability of Default Models

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ABSTRACT

Banks and Companies play an important role in the entire world economy. On December, 2009, the Basel Committee on Banking Supervision has proposed a package to strengthen global capital and liquidity regulations with the goal of promoting a more resilient banking and economic sector. The committee proposed a series of measures to promote the buildup of capital buffers in good times that can be drawn upon in periods of stress. It would be recalled that the Probability of Default is the major component when determining (i) Capital Requirements under Basel II (Now Basel III) (ii) Expected Loss (iii) Risk Weighted Asset (iv) Request of Loans (v) Rating Estimation (vi) Pricing of Credit Derivatives. The false estimation of probability of default leads to unreasonable rating; incorrect pricing of financial instruments and thereby it was one of the causes of the recent global financial crises.

However, in an attempt to promote the building of capital buffers in good times that can be drawn upon in periods of stress, author consider the most effective and commonly used probability of default models (Logit and Probit) as reference research models. The modelling of the existing Logit and Probit models did not incorporate any financial crisis component right from the inception and so they tends to underestimates (or overestimates) the probability of the adverse out comes.

In this research paper, with reference to the existing Logit and Probit models, author proposed about TWENTY (20) Advanced Stressed Probability of Default Models which validly incorporated financial crisis components and addressed the problem of underestimating (or overestimating) the probability of default during stress periods. The TWENTY (20) Banking and Economic Advanced Stressed Probability of Default Models were found working and provide much closed approximations to that of existing Logit and Probit, in the same vein captured "fat – tailed effects", which are not being captured by traditional normal distribution. Another aim of this paper is to make an attempt to predict Black Swan Events.

Author considers financial data of five (5) U.S. based companies listed on the platform of New York Stock Exchange (NYSE) from 2014 – 1991 (25 years) inclusive and extracted from Yahoo Finance.

KEYWORDS: Stress Test, Probability of Default, Logit, Probit, Log – Logistic (3P), JAMEEL

Introduction

The Global financial markets have experienced series of financial and economic crises right from the inception and from generation to generation. Banks, Companies and the world economy experienced catastrophic deterioration and serious corporate failures by systemic risk effect.

Big Banks and Companies like Continental Illinois, City Federal Savings and Loan, Bank of New England Boston, Lehman Brothers, General Motors, and Worldcom have all failed and declared bankrupt in the history of global financial markets. That is why, many scholars in the past and recent past have attempted to comes up with the models that can precisely calculate the **Probability of Default** of a Bank or Company over a given time period. Probability of Default for a given Company or Bank captures the probability that the Company or Bank will default within a certain period.

The most popular models used by financial institutions to calculate probabilities of default are **LOGIT (1980)** and **PROBIT (1981)**. Despite the fact that Logit and Probit gives good approximations but seems not to capture chaotic markets behavior to some extent. Accurately determination of probability of default play very important role in the entire world economy. Probability of Default is the major component when determining (i) Capital Requirements under Basel II (Now Basel III) (ii) Expected Loss (iii) Risk Weighted Asset.

Also, the probability of default (PD) is a crucial parameter in risk management, and can be used for the **requests of loans, rating estimation, pricing of credit derivatives and many others key financial fields.** The false estimation of PD leads to **unreasonable rating; incorrect pricing of financial instruments and thereby it was one of the causes of the recent global financial crisis.**

The aim of this research work is to come up with new advanced stressed probability models that can capture chaotic markets behaviour or in the other way round to work under financial markets distress to some extent.

However, with reference to most popular Probability of Default Models: Logit and Probit, Author humbly developed **TWEENTY (20)** Advanced Stressed Probability of Default Models that can estimates probability of default in similar way that Traditional Logit and Probit Models can estimate or even better. Because these **TWEENTY (20)** models are expected to work even during financial crisis, since, the crisis components are incorporated into the models.

Author therefore considered **FIVE (5)** international Companies for this research work:

(1) **Microsoft Corporation (MSFT)**
(2) **Exxon Mobil (XOM)**
(3) **Chevron Corporation (CVX)**
(4) **Honda Motor Corporation (HMC)**
(5) **General Electric (GE)**

MATERIAL AND METHODS

The methodology adopted in this research work is to consider the two (2) fundamental probabilities of default models namely: **LOGIT** and **PROBIT** and to develop new advanced stressed probabilities of default models that can capture **LOW - PROBABILITY, HIGH – IMPACT** events popularly known as **BLACK SWAN** events by incorporating crises components into the Existing **LOGIT** and **PROBIT** as follows:

Models Assumptions:

(i) I assume that the Arithmetic Means of the Companies' Stock Return and the Macroeconomic indicators are all non – negative (positive)

(ii) I assume Geometric Mean and Volatility $(\mu_A \ and \ \sigma_A)$ of Arithmetic Means and volatilities of the Macroeconomic Indicators including Stock Return instead of Arithmetic Mean

Jameel's Models I:

The proposed models considering simple Logistic Regression Model are given by:

Type A:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{K} \beta_i X_i \right) \mp \sigma_A f\left(x; \mu_{company}, \sigma_{company}, \xi\right)}$$

Type B:

$$PD_{Stressed} = \frac{1}{1 + \exp \mu_A \left(\sum_{i=0}^{K} \beta_i X_i \right) \mp f\left(x; \mu_{company}, \sigma_{company}, \xi\right)}$$

Type C:

$$PD_{Stressed} = \frac{1}{1 + \exp \left(\sum_{i=0}^{K} \beta_i X_i \right) \mp \sigma_A f\left(x; \mu_{company}, \sigma_{company}, \xi\right)}$$

Type D:

$$PD_{Stressed} = \frac{1}{1 + \exp \left(\sum_{i=0}^{K} \beta_i X_i \right) \mp f\left(x; \mu_{company}, \sigma_{company}, \xi\right)}$$

Where, the **Logistic Regression Model (LOGIT)** is

given by:

$PD = \frac{1}{1 + \exp \left(\sum_{i=0}^{K} \beta_i X_i \right)}$, $X = (X_1, X_2, \ldots, X_k)$ is a vector of explanatory variables (Macro-economic Indicators). And $f\left(x, \mu_{company}, \sigma_{company}, \xi\right)$ is the best fat – tailed effects probability distribution of the underlying asset return; μ_A is the Geometric Return of the Arithmetic Means of the U.S. Macroeconomic Indicators including the research Company Stock Return; σ_A is the Geometric Volatility of the Volatilities of the U.S. Macroeconomic Indicators including the research Company Stock Return.

Also, note that the Macroeconomic Indicators (Independent Variables) depends on the **OPTION/FUTURES OR STOCK EXCHANGES (COUNTRIES)** in which the **Research Company** has being listed.

Survival Functions: For example, the Survival functions of a Company under stress using $M1\ TYPE\ A$ are given by:

$$PD_{Stressed}\left(Survival\right) = 1 - PD_{Stressed} = \frac{\exp\left(\mu_A \sum_{i=1}^{k} \beta_i X_i\right) \mp \sigma_A f\left(x; \mu, \sigma, \xi\right)}{1 + \exp\left(\mu_A \sum_{i=1}^{k} \beta_i X_i\right) \mp \sigma_A f\left(x; \mu, \sigma, \xi\right)}$$

In similar way, we can find the survival functions of the remaining types.

The **Partial Change in Stressed Probability of Default** reference to **M1 TYPE A** is given by:

$$\partial PD_{Stressed} = \left(\frac{\pm \sigma_A \dfrac{\partial f\left(x; \mu_{company}, \sigma_{company}, \xi\right)}{\partial x}}{\left[1 + \exp \mu_A \left(\sum_{i=0}^{K} \beta_i X_i \right) \mp \sigma_A f\left(x; \mu_{company}, \sigma_{company}, \xi\right) \right]^2} \right) \times \partial x$$

In similar way, we can find the other Partial Derivatives of the remaining types.

Jameel's Models II:

The proposed models considering Merton's Model are given by:

Type A:

$$PD_{Stressed} = \Phi\left(\beta_0 + \mu_A \sum_{j=1}^{J} \beta_j X_j \right) \pm \sigma_A f\left(x; \mu_{company}, \sigma_{company}, \xi\right)$$

Type A*:

$$PD_{Stressed} = \Phi\left[\mu_A \left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j \right) \right] \pm \sigma_A f\left(x; \mu_{company}, \sigma_{company}, \xi\right)$$

Type B:

$$PD_{Stressed} = \Phi\left(\beta_0 + \mu_A \sum_{j=1}^{J} \beta_j X_j \right) \pm f\left(x; \mu_{company}, \sigma_{company}, \xi\right)$$

Type B*:

$$PD_{Stressed} = \Phi\left[\mu_A \left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j \right) \right] \pm f\left(x; \mu_{company}, \sigma_{company}, \xi\right)$$

Type C:

$$PD_{Stressed} = \Phi\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j \right) \pm \sigma_A f\left(x; \mu_{company}, \sigma_{company}, \xi\right)$$

Type D:

$$PD_{Stressed} = \Phi\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j \right) \pm f\left(x; \mu_{company}, \sigma_{company}, \xi\right)$$

Where, **Factor Model based on Merton Model (PROBIT)** is given:

$$PD = \Phi\left(\beta_0 + \sum_{j=1}^{J} \beta_j X_j \right)$$

For example, the Survival functions of a Company under stress using $M2\ TYPE\ A$ are given by:

$$PD_{Stressed}\left(Survival\right) = 1 - PD_{Stressed} = 1 - \left[\Phi\left(\beta_0 + \mu_A \sum_{j=1}^{J} \beta_j X_j \right) \pm \sigma_A f\left(x; \mu_{company}, \sigma_{company}, \xi\right) \right]$$

In similar way, we can find the survival functions of the remaining types.

The **Partial Change in Stressed Probability of Default** reference to **M2 TYPE A** is given by:

$$\partial PD_{Stressed} = \left[\pm \sigma_A \frac{\partial f\left(x; \mu_{company}, \sigma_{company}, \xi\right)}{\partial x} \right] \times \partial x$$

In similar way, we can find the other Partial Derivatives of the remaining types.

Some Selected Data Sources: Yahoo Finance, Google Finance, Federal Reserve Bank , Economic Research

Companies and Fundamental Macroeconomic Indicators used in the Research Work:

In this research work, I consider:

(a) Five (5) companies listed on the platform of New York Stock Exchange (NYSE) namely; Chevron Corporation, Honda Motor Corporation, Microsoft Corporation, Exxon Mobil Corporation, and General Electric Corporation for the period of Twenty Five (25) years (1991 – 2014) data
(b) The stock returns of the five (5) companies under consideration
(c) The U.S. GDP
(d) The U.S. Inflation Rate
(e) The U.S. Prime Rate
(f) The U.S. unemployment Rate
(g) The U.S. USD/GBP Exchange Rate
(h) The U.S. House Price
(i) The U.S. Oil Price
(j) The U.S. Gold Price

Using **QI Macros 2015 Software**, I obtained the following components :

Multiple Regression Model Component of CHEVRON Corporation (CVX) for calculating Probability of Default:

$$\begin{aligned} Y_{CHEVRON} = {} & 0.004 + 0.004 \times \Delta P(CHEVRON) - 0.199 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.009 \times \Delta P(INF) \\ & - 0.018 \times \Delta P(UER) + 0.002 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD/GBP) \end{aligned}$$

Multiple Regression Model Component of GENERAL ELECTRIC(GE) for calculating Probability of Default:

$$\begin{aligned} Y_{GE} = {} & 0.004 - 0.001 \times \Delta P(GE) - 0.207 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.016 \times \Delta P(INF) \\ & - 0.017 \times \Delta P(UER) - 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD/GBP) \end{aligned}$$

Multiple Regression Model Component of MICROSOFT (MSFT) Corporation for calculating Probability of Default:

$$\begin{aligned} Y_{MSFT} = {} & 0.004 - 0.006 \times \Delta P(MSFT) - 0.189 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.011 \times \Delta P(INF) \\ & - 0.017 \times \Delta P(UER) + 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD/GBP) \end{aligned}$$

Multiple Regression Model Component of EXXON MOBIL (XOM) Corporation for calculating Probability of Default:

$$\begin{aligned} Y_{XOM} = {} & 0.004 + 0.002 \times \Delta P(XOM) - 0.2 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.01 \times \Delta P(INF) \\ & - 0.018 \times \Delta P(UER) + 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD/GBP) \end{aligned}$$

Multiple Regression Model Component of HONDA MOTOR CO., Ltd for calculating Probability of Default:

$$\begin{aligned} Y_{HONDA} = {} & 0.004 - 0.004 \times \Delta P(HMC) - 0.204 \times \Delta P(GDP) + 0.009 \times \Delta P(OIL) + 0.01 \times \Delta P(INF) \\ & - 0.018 \times \Delta P(UER) + 0.001 \times \Delta P(GOLD) + 0 \times \Delta P(INTEREST) + 0 \times \Delta P(USD/GBP) \end{aligned}$$

Merely introduction of Probability Distribution Function in the Proposed **Jameel Models I and II** is not enough; we need to runs the test of Goodness of fit of the Probability Distribution to obtain the **Best fitted Probability Distribution.**

TEST OF GOODNESS OF FIT OF THE FIVE (5) COMPANIES' STOCKS RETURNS

In this test of Goodness of fit, author consider Five (5) criteria:

(i) Author accept if the Average of the ranks of Kolmogorov Smirnor, Anderson Darling and Chi-squared is less than or equal to Three (3)

(ii) Author must choose the Probability Distribution follows by the data **ITSELF** regardless of its Rankings

(iii) If there is tie, we include both Probability Distributions in the selection

(iv) At least Two (2) probability distributions must be included in the selection

(v) Author selects the most occur probability distribution as the qualify candidate in each case of test of goodness of fit of the stock returns.

The following tables provide the summary results of the test of goodness of fit of the research companies:

Name of Probability Distribution (Chevron)	Kolmogorov Smirnov Rank	Anderson Darling Rank	Chi-Squared Rank	Average
Burr	3	3	2	2.07
Dagum	2	1	1	1.33
Log - Logistic(3P)	1	2	3	2.0

Name of Probability Distribution (Honda)	Kolmogorov Smirnov Rank	Anderson Darling Rank	Chi-Squared Rank	Average
Burr	1	2	1	1.33
Dagum	2	3	2	2.33
Log - Logistic(3P)	3	1	3	2.33

Name of Probability Distribution (Microsoft)	Kolmogorov Smirnov Rank	Anderson Darling Rank	Chi-Squared Rank	Average
Dagum	1	1	1	1.0
Log - Logistic(3P)	2	4	4	3.33

Name of Probability Distribution (Exxon Mobil)	Kolmogorov Smirnov Rank	Anderson Darling Rank	Chi-Squared Rank	Average
Burr	3	3	3	3.00
Dagum	2	2	4	2.67
Log - Logistic(3P)	1	1	1	1.00

Name of Probability Distribution (General Electric)	Kolmogorov Smirnov Rank	Anderson Darling Rank	Chi-Squared Rank	Average
Error	3	3	2	2.67
Laplace	2	2	3	2.33
Log – Logistic (3P)	1	1	1	1.0

Hence, the BEST fitted fat – tailed Probability Distribution is the **LOG – LOGISTIC (3P)** and is given by:

$$f(x;\mu,\sigma,\xi)=\begin{cases}\dfrac{\left(1+\dfrac{\xi(x-\mu)}{\sigma}\right)^{-\left(\frac{1}{\xi}+1\right)}}{\left[1+\left(1+\dfrac{\xi(x-\mu)}{\sigma}\right)^{-\frac{1}{\xi}}\right]^{2}} & ;\quad x\geq\mu\\ 0 & ;\quad elsewhere\ (x\leq\mu)\end{cases}$$

is called Generalized Log-Logistic or Log-Logistic (3P) Probability Distribution. Where, $\mu\in\mathbb{R}$ is the location parameter, $\sigma>0$ the scale parameter and $\xi\in\mathbb{R}$ the shape parameter. The shape parameter ξ is often restricted to lie in $[-1,1]$, when the probability density function is bounded.

However, in this research work author will restrict ξ to only $\xi=1$. However, one can test, for $\xi=0$ or $\xi=-1$ and other values on $[-1,1]$ in the subsequent researchers. Log – Logistic (3P) is similar in shape to the <u>**log-normal distribution**</u> **but has** <u>**heavier tails**</u>**.**

Also, using our **data sources, Microsoft EXCEL and QI Macros 2015,** author obtained the following:

Geometric Mean and Volatility (Standard Deviation) of the Macroeconomic indicators plus stock return for **Chevron Corporation** are given by: $\mu_{GEO}(Chevron)=0.030383975$ and $\sigma_{GEO}(Chevron)=0.111414539$.

Arithmetic Mean and Volatility (Standard Deviation) of **Chevron Stock Return** are given by: $\mu_{STOCK}(Chevron)=0.004402791$ and $\sigma_{STOCK}(Chevron)=0.06909299$.

Geometric Mean and Volatility (Standard Deviation) of the Macroeconomic indicators plus stock return used for **Honda Motor Corporation** are given by: $\mu_{GEO}(Honda)=0.031352397$ and $\sigma_{GEO}(Honda)=0.114001187$

Arithmetic Mean and Volatility (Standard Deviation) of **Honda Motor Stock Return** are given by: $\mu_{STOCK}(Honda)=0.005839335$ and $\sigma_{STOCK}(Honda)=0.084945727$.

Geometric Mean and Volatility (Standard Deviation) of the Macroeconomic indicators plus stock return used for **Microsoft Corporation** are given by: $\mu_{GEO}(MSFT)=0.031352397$ and $\sigma_{GEO}(MSFT)=0.117906073$

Arithmetic Mean and Volatility (Standard Deviation) of **Microsoft Corporation Stock Return** are given by: $\mu_{STOCK}(MSFT)=0.006798657$ and $\sigma_{STOCK}(MSFT)=0.115022493$

Geometric Mean and Volatility (Standard Deviation) of the Macroeconomic indicators plus stock return used for **Exxon Mobil Corporation** are given by: $\mu_{GEO}(XOM) = 0.030729517$ and $\sigma_{GEO}(XOM) = 0.110236167$.

Arithmetic Mean and Volatility (Standard Deviation) of **Exxon Mobil Corporation Stock Return** are given by: $\mu_{STOCK}(XOM) = 0.00487448$ and $\sigma_{STOCK}(XOM) = 0.062787634$.

Geometric Mean and Volatility (Standard Deviation) of the Macroeconomic indicators plus stock return used for **General Electric Corporation** is given by: $\mu_{GEO}(GE) = 0.037067141$ and $\sigma_{GEO}(GE) = 0.10009902$

Arithmetic Mean and Volatility (Standard Deviation) of **General Electric Corporation Stock Return** is given by: $\mu_{STOCK}(GE) = 0.002163529$ and $\sigma_{STOCK}(GE) = 0.091140157$

RESULT AND DISCUSSION

Having completed the test of goodness of fit and found that the Best fitted fat tailed probability distribution is **LOG – LOGISTIC (3P)** and the values of $\mu_{company}$, $\sigma_{company}$, μ_A and σ_A of each of the research company are also computed then author is now ready to implement and present the Results of the proposed **Jameel's I and II models**, discuss them and forward a new research Theorem and some corollaries derived from them.

Under **Chevron Corporation, on the month of June, 2014,** the probability of default using the **existing LOGIT is 0.499097747%** and that of **PROBIT is 0.501439786%, whereas for JAMEEL'S PROPOSED MODELS I AND II are respectively: 0.499976914%, 0.499968258%, 0.500011436%, 0.499933742%, 0.49910206%, 0.499093434%, 0.499136461%, 0.499059039%, 0.501573711%, 0.50160834%, 0.501435622%, 0.501746429%, 0.501422471%, 0.5014571%, 0.501284382%, and 0.501595189%**. While in the case of **M2 TYPE A*** and **M2 TYPE B*,** on 10/1/2014, Author obtained the stressed probabilities: **0.500003829, 0.50000384, 0.500003782, and 0.500003887** which are clearly lies in between the probabilities of **LOGIT** and **PROBIT**. This gives birth to a new **THEOREM** that indicates the existence of some new advanced probability functions **(Jameel's functions)** in between the traditional **LOGIT** and **PROBIT** models and shall be discussed below.

Under **Honda Motor, on the month of December, 2014,** the probability of default using the **existing LOGIT is 0.49898654%** and that of **PROBIT is 0.501617235%, whereas for JAMEEL'S PROPOSED MODELS I AND II are respectively: 0.49996827%, 0.499968182%, 0.49996784%, 0.49896591%, 0.498986503%, 0.498986932%, 0.498986163%, 0.501594751%, 0.501595103%, 0.501593383%, 0.50159647%, 0.501617059%, 0.50161741%, 0.501615691%, and 0.501618778%.** While in the case of **M2 TYPE A*** and **M2 TYPE B*,** on **12/1/2014**, Author obtained the stressed probabilities: **0.50005052, 0.50005088, 0.500043161, and 0.500052246** which are clearly lies in between the probabilities of **LOGIT** and **PROBIT**.

Under **Microsoft Corporation, on the month of December, 2014,** the probability of default using the **existing LOGIT is 0.499506722%** and that of **PROBIT is 0.500787158%, whereas for JAMEEL'S PROPOSED MODELS I AND II are respectively: 0.499984689%, 0.499983853%, 0.499987817%, 0.499980725%, 0.499507139%, 0.499506304%, 0.49951026%, 0.499503183%, 0.50161708%, 0.501620425%, 0.501604569%, 0.501632936%, 0.500785486%, 0.50078831%, and 0.500772975%, 0.500801342%.** While in the case of **M2 TYPE A*** and **M2 TYPE B*,** on **8/1/2014**, Author obtained the stressed probabilities: **0.500006411, 0.50004996, 0.49984351, and 0.500212861** which are clearly lies in between the probabilities of **LOGIT** and **PROBIT**.

Under **Exxon Mobil, on the month of October, 2014,** the probability of default using the **existing LOGIT is 0.499779809%** and that of **PROBIT is 0.500351375%, whereas for JAMEEL'S PROPOSED MODELS I AND II are respectively: 0.499996206%, 0.499994665%, 0.500002424%, 0.4999988447%, 0.499780578%, 0.499779039%, 0.499786791%, 0.499772826%, 0.5015668886%, 0.501573051%, 0.501542014%, 0.501597925%, 0.500348293%, 0.500354456%, 0.500323419%, and 0.500379339%** respectively. While in the case of **M2 TYPE A*** and **M2 TYPE B*,** on **6/2/2014**, Author obtained the stressed probabilities: **0.500028154, 0.500028318, 0.500027492, and 0.500028373** which are clearly lies in between the probabilities of **LOGIT** and **PROBIT** obtained above.

Under **General Electric, on the month of September, 2014,** the probability of default using the **existing LOGIT is 0.499256894%** and that of **PROBIT is 0.501185825%, whereas for JAMEEL'S PROPOSED MODELS I AND II are respectively: 0.499973123%, 0.499971787%, 0.499979127%, 0.499965783%, 0.49925736%, 0.499256228%, 0.499263546%, 0.499250241%, 0.501608698%, 0.501614042%, 0.50158468%, 0.50163806%, 0.501183154%, 0.501188497%, 0.501159135%, and 0.501212515%.** While in the case of **M2 TYPE A*** and **M2 TYPE B*,** on **3/2/2014**, Author obtained the stressed probabilities: **0.500041284, 0.500046627, 0.500017265, and 0.500070645** which are clearly lies in between the probabilities of **LOGIT** and **PROBIT** obtained above. This gives birth to a new **THEOREM** that indicates the existence of some new advanced probability functions **(Jameel's functions)** in between the traditional **LOGIT** and **PROBIT** models and shall be discussed later.

PROPOSED THEOREM (JAMEEL'S THEOREM)

Let $L(X)$ be a **Logistic Regression model** of default probability, let $P(X)$ be a **Probit model**, let $X = (X_1, X_2, X_3, \ldots, X_k)$ be a vector of independent variables then there exists some **Jameel's Banking**

and Economic Advanced Stressed Probability of Default Models $J_1(X), J_2(X), and\ J_3(X)$ such that

$0 \le J_1(X) \le L(X) \le J_2(X) \le P(X) \le J_3(X) \le 1$.

Where the general form of the functions are:

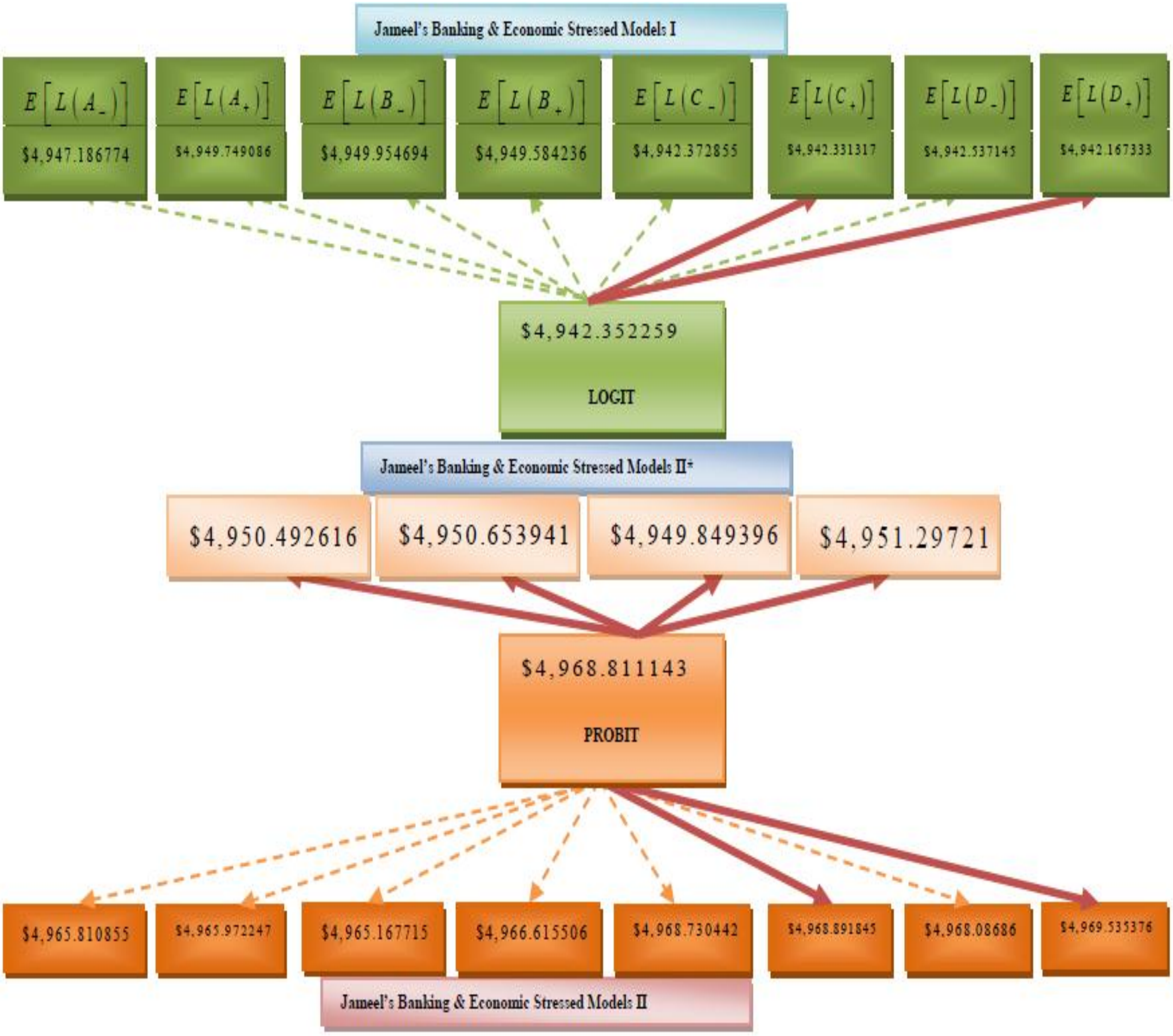
$J_1(X) = \mu_A L(X) \pm \sigma_A f(x, \mu, \sigma, \xi)$,

$J_2(X) = \mu_A [P(X)] \pm \sigma_A f(x, \mu, \sigma, \xi)$, and

$J_3(X) = \mu_A P(X) \pm \sigma_A f(x, \mu, \sigma, \xi)$. x is the daily, monthly or annually stock return and $x \ge \mu$ (mean of the stock return). $\mu_A\ and\ \sigma_A$ are the Geometric Mean and Volatility of the independent variables plus stock return and $f(x, \mu, \sigma, \xi)$ is a Log – Logistic (3P) probability distribution or any other best fitted fat – tailed probability distribution.

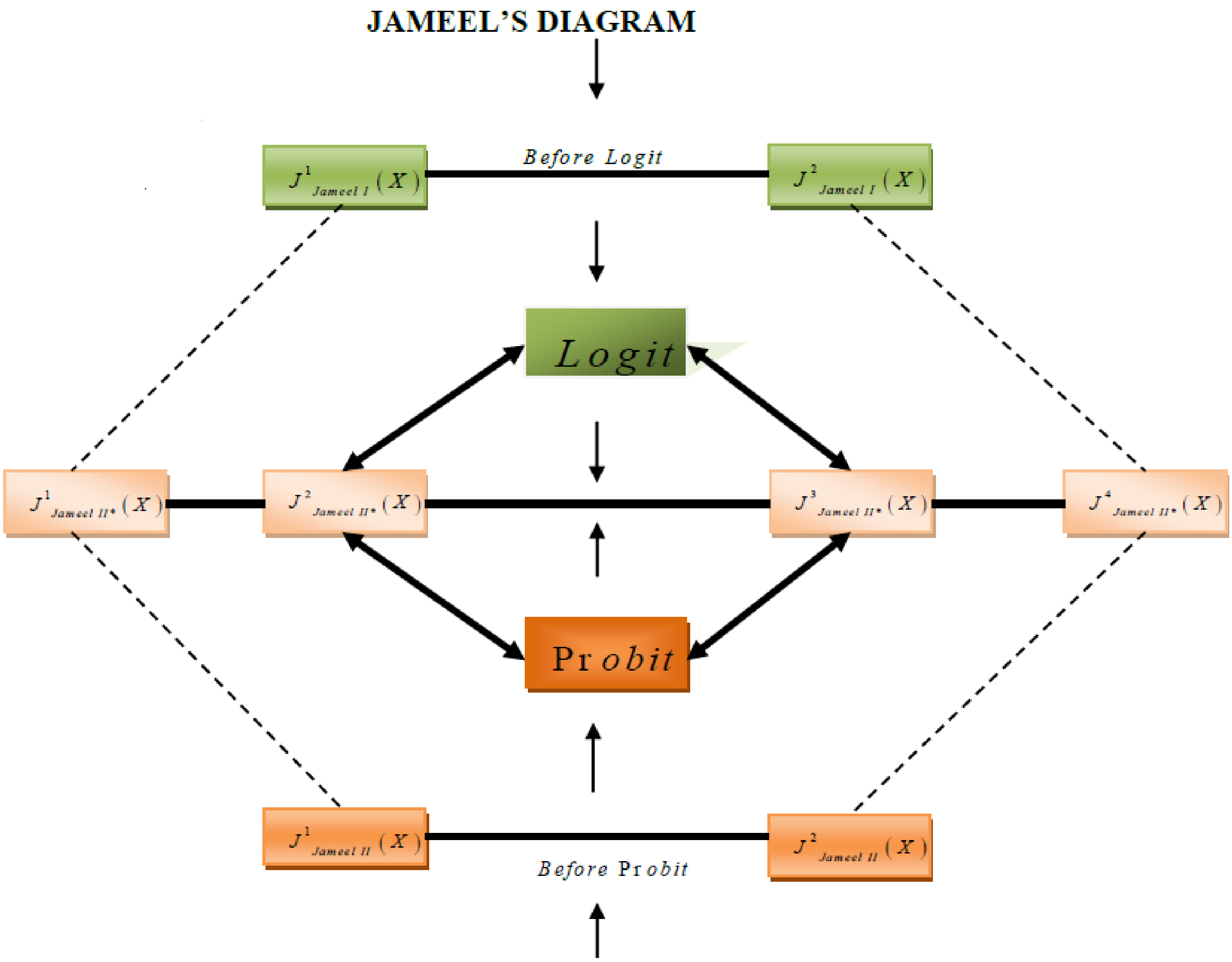
Example:

Let E_i be the **Exposure at Default**, P_i be the **Probability of Default** and L_i be the **Loss given Default** then the **Expected Loss** is given by : $E[L] = P_i \times E_i \times L_i \quad for\ \ i = 1, 2, \ldots, 20$. Using the **Chevron Corporation Stressed Probabilities of Default (on 12/1/2014)** and **Stressed Beta Recovery Rate** Formula, where $E_i = 1 - R_i$ for each i. Assume using the **Foundation or Advanced Approach under BASEL II**, we obtained the value of the Exposure at Default $EAD = \$1,000,000$ then the **Stressed Expected Loss** using the **Jameel's Banking and Economic Advanced Stressed Probability of Default Models** can be illustrated in the following figure:



All the functions indicated in the **red bold continuous line**, satisfied the inequality in the **JAMEEL'S THEOREM** above. This clearly shows the existence of **JAMEEL'S THEOREM.**

JAMEEL'S DIAGRAM



Jameel's Diagram also proved the existence of **Jameel's Theorem**.

MORE RESULTS IN THE RESEARCH STUDY

(1) In this Research Work, Author use only , of course, we can perform what we called Simulation Analysis using Different values of on the interval to obtain different Advanced Stressed Probabilities of Default

(2) Based on this Research Work, Dagum Probability Distribution is another very good candidate to be used in our Proposed Advanced Stress Probability of Default Models

(3) Burr, Extreme Probability Distributions are also useful Candidates

(4) The Proposed Models can also be used in other fields of Sciences, Social Sciences and Engineering to capture chaotic situations

(5) PROPOSED COROLLARIES DERIVED FROM THE RESEARCH WORK

Linear Discriminant Analysis:

Linear Discriminant Analysis can be used to produce a direct estimate of the probability of default. The

Company's probability of default is given by:

$$PD = \frac{1}{\left(1 + \frac{(1-\pi_B)}{\pi_B} e^{z_i - \alpha}\right)}$$

Corollary 1 (Proposed Jameel's Models III):

TYPE A:

$$PD_{Stressed} = \frac{1}{\left(1 + \frac{(1-\pi_B)}{\pi_B} e^{\mu_A (z_i - \alpha)}\right) \pm \sigma_A f(x, \mu, \sigma, \xi)}$$

TYPE B:

$$PD_{Stressed} = \frac{1}{\left(1 + \frac{(1-\pi_B)}{\pi_B} e^{\mu_A (z_i - \alpha)}\right) \pm f(x, \mu, \sigma, \xi)}$$

TYPE C:

$$PD_{Stressed} = \frac{1}{\left(1 + \frac{(1-\pi_B)}{\pi_B} e^{(z_i - \alpha)}\right) \pm \sigma_A f(x, \mu, \sigma, \xi)}$$

TYPE D:

$$PD_{Stressed} = \frac{1}{\left(1 + \frac{(1-\pi_B)}{\pi_B} e^{(z_i - \alpha)}\right) \pm f(x, \mu, \sigma, \xi)}$$

Existing Model (Multinomial Logistic Regression):

$$P(Y_i = K) = \frac{1}{1 + \sum_{K=1}^{K-1} \exp(\beta_K X_K)}$$

TYPE A:

$$PD_{Stressed} = P(Y_i = K) = \frac{1}{1 + \sum_{K=1}^{K-1} \exp \mu_A (\beta_K X_K) \pm \sigma_A f(x, \mu, \sigma, \xi)}$$

TYPE A* (Higher Probabilities):

$$PD_{Stressed} = P(Y_i = K) = \frac{1}{1 + \mu_A \sum_{K=1}^{K-1} \exp(\beta_K X_K) \pm \sigma_A f(x, \mu, \sigma, \xi)}$$

TYPE B:

$$PD_{Stressed} = P(Y_i = K) = \frac{1}{1 + \sum_{K=1}^{K-1} \exp \mu_A (\beta_K X_K) \pm f(x, \mu, \sigma, \xi)}$$

TYPE B*(Higher Probabilities):

$$PD_{Stressed} = P(Y_i = K) = \frac{1}{1 + \mu_A \sum_{K=1}^{K-1} \exp(\beta_K X_K) \pm f(x, \mu, \sigma, \xi)}$$

TYPE C:

$$PD_{Stressed} = P(Y_i = K) = \frac{1}{1 + \sum_{K=1}^{K-1} \exp(\beta_K X_K) \pm \sigma_A f(x, \mu, \sigma, \xi)}$$

TYPE D:

$$PD_{Stressed} = P(Y_i = K) = \frac{1}{1 + \sum_{K=1}^{K-1} \exp(\beta_K X_K) \pm f(x, \mu, \sigma, \xi)}$$

Existing Model (Instantaneous Probability of Default Model):

$$P(0,T) = \exp\left\{-\int_0^T \lambda(t)\,dt\right\}$$

Where $\lambda(t)$ is called Default Hazard Rate Function for all $t \geq 0$. $P(0,T)$ is the Probability of Survival and the Probability of a Default between time 0 and time T will thus be $1 - P(0,T)$.

Corollary 3 (Proposed Jameel's Models V):

TYPE A:

$$PS_{Stressed} = \exp\left\{-\int_0^T \mu_A \lambda(t)\,dt\right\} \pm \sigma_A \int_0^T f(t, \mu, \sigma, p)\,dt$$

TYPE A* (Higher Probabilities):

$$PS_{Stressed} = \mu_A \exp\left\{-\int_0^T \lambda(t)\,dt\right\} \pm \sigma_A \int_0^T f(t, \mu, \sigma, p)\,dt$$

TYPE B:

$$PS_{Stressed} = \exp\left\{-\int_0^T \mu_A \lambda(t)\,dt\right\} \pm \int_0^T f(t, \mu, \sigma, p)\,dt$$

TYPE B*(Higher Probabilities):

$$PS_{Stressed} = \exp\left\{-\int_0^T \mu_A \lambda(t)\,dt\right\} \pm \int_0^T f(t, \mu, \sigma, p)\,dt$$

TYPE C:

$$PS_{Stressed} = \exp\left\{-\int_0^T \lambda(t)\,dt\right\} \pm \sigma_A \int_0^T f(t, \mu, \sigma, p)\,dt$$

TYPE D:

$$PS_{Stressed} = \exp\left\{-\int_0^T \lambda(t)\,dt\right\} \pm \int_0^T f(t, \mu, \sigma, p)\,dt$$

Existing Model (Mixed Logit Model):

$$P = \int_{-\infty}^{\infty} P(\beta, x_i; v_i)\, N(\beta/\mu, \sigma)\, d\beta$$

Corollary 4 (Proposed Jameel's Models VI):

TYPE A:

$$PD_{STRESSED} = \int_{-\infty}^{\infty} P(\beta, x_i; v_i)\, N\{\mu_A(\beta/\mu, \sigma)\}\, d\beta \pm \sigma_A \int_0^{\infty} f(\beta, \mu^*, \sigma^*, p)\, d\beta$$

TYPE A* (Higher Probabilities):

$$PD_{STRESSED} = \mu_A \int_{-\infty}^{\infty} P(\beta, x_i; v_i)\, N(\beta/\mu, \sigma)\, d\beta \pm \sigma_A \int_0^{\infty} f(\beta, \mu^*, \sigma^*, p)\, d\beta$$

TYPE B:

$$PD_{STRESSED} = \int_{-\infty}^{\infty} P(\beta, x_i; v_i)\, N\{\mu_A(\beta/\mu, \sigma)\}\, d\beta \pm \int_0^{\infty} f(\beta, \mu^*, \sigma^*, p)\, d\beta$$

TYPE B*(Higher Probabilities):

$$PD_{STRESSED} = \mu_A \int_{-\infty}^{\infty} P(\beta, x_i; v_i)\, N(\beta/\mu, \sigma)\, d\beta \pm \int_0^{\infty} f(\beta, \mu^*, \sigma^*, p)\, d\beta$$

TYPE C:

$$PD_{STRESSED} = \int_{-\infty}^{\infty} P(\beta, x_i; v_i)\, N(\beta/\mu, \sigma)\, d\beta \pm \sigma_A \int_0^{\infty} f(\beta, \mu^*, \sigma^*, p)\, d\beta$$

TYPE D:

$$PD_{STRESSED} = \int_{-\infty}^{\infty} P(\beta, x_i; v_i)\, N(\beta/\mu, \sigma)\, d\beta \pm \int_0^{\infty} f(\beta, \mu^*, \sigma^*, p)\, d\beta$$

Corollary 5 (Proposed):
Any Probability of Default Model can be stressed using the Methodology in **Corollary 1, Corollary 2, Corollary 3, and Corollary 4** above.

RECOVERY RATE

$f(x) = c.x^{a}(1-x)^{b}$ is called Beta Probability Distribution where $0 \le x \le 1$.

$a = \frac{\mu^{2}(1-\mu)}{\sigma^{2}}$, $b = \frac{\mu(1-\mu)^{2}}{\sigma^{2}}$. μ is the mean of the data, and σ is its standard deviation.

Corollary 6 (Proposed Jameel's Models VII):

TYPE A: $c\left(x^{a}(1-x)^{b}\right)^{\mu_A} \pm \sigma_A f(x;\mu,\sigma,\xi)$, **TYPE B:** $c\left(x^{a}(1-x)^{b}\right)^{\mu_A} \pm f(x;\mu,\sigma,\xi)$

TYPE C: $c\left(x^{a}(1-x)^{b}\right) \pm \sigma_A f(x;\mu,\sigma,\xi)$, **TYPE D:** $c\left(x^{a}(1-x)^{b}\right) \pm f(x;\mu,\sigma,\xi)$

Corollary 7 (Proposed):
Any Recovery Rate Model can be stressed using the Methodology in **Corollary 6** above.

CONCLUSION

The proposed **Jameel's** Banking and Economic Advanced Stressed Probability of Default models have the ability to:

(1) Detects **PAST** Probabilities of Default, Economic and Financial Crises given accurate, reliable and valid past Economic and Financial Data
(2) Predicts **FUTURE Probabilities of Default, Economic and Financial Crises** given accurate, reliable and valid forecast of the independent variables (macroeconomic indicators) plus stock return.

Note that medium and long term forecasts of key economic indicators for G20 countries are estimated by major international organizations namely; **World Bank, IMF, United Nations, OECD, European Commission and the Economist Intelligence Unit.**

Also, we can forecast any company stock price (return) using sophisticated softwares like **VantagePoint, GMDH Shell, AptiStock, StockwareLite, GoldenGem and A2Scanner.**

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