RESEARCH ARTICLE

**Received on: 14-10-2014**
**Accepted on: 15-02-2015**
**Published on: 26-02-2015**

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Conflict of Interest: None Declared !

# The maximum difference method to find initial basic feasible solution for transportation problem

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## ABSTRACT

In industries, raw material (finished products) is transported from factories to ware houses or vice-versa, which involves transportation cost. To optimize this cost and hence reduce the total cost of finished product, transportation problem is used. In Mathematics and Economics, transportation theory is the name given to the study of optimal transportation and allocation of resources. The transportation problem is a special case of Linear Programming Problem which deals with the distribution of single commodity from various sources of supply to various destinations of demand in such a manner that the total transportation cost is minimized. The existing methods to find initial basic feasible solutions are North West-Corner Method (NWCM), Least Cost Method (LCM) and Vogel's Approximation Method (VAM). In this paper a new method is proposed for finding an initial basic feasible solution of a transportation problem. The method discussed in this paper is named as Maximum Difference Method. This method gives an initial basic feasible solution of the transportation problem, which is most of the time better than that of Vogel's Approximation Method (VAM). The proposed algorithm is illustrated using some numerical examples.

**KEYWORDS:** IBFS, Optimal Solution, Transportation Problem, VAM..

## INTRODUCTION

Transportation Problem is a special case of Linear Programming Problem which deals with the distribution of single commodity from various sources of supply to various destinations of demand in such a manner that the total transportation cost is minimized. Transportation problem was firstly presented by F.L.Hitchcock[1], in his paper "The distribution of a product from several sources to numerous localities" and after that T.C. Koopmans [2], presented in his historic paper "Optimum utilization of the transportation system". These two papers are the milestones in the development of the various methods to solve the transportation problem.The initial basic feasible solution of transportation problem are obtained by North-west corner method (NWCM), Least cost method (LCM) and Vogel's Approximation Method (VAM) [Reinfeld and Vogel][3]. Transportation problem was further developed by Dantzig[4]. In 1984, S.K.Goyal [5] presented a paper, in which improved version of VAM was used for unbalanced transportation problem. Kirca and Stair [6] developed a heuristic method to obtain an efficient initial basic feasible solution.

The transportation algorithm is based on the assumption that the model is balanced, which means that the total demand is equal to the total supply. If the model is unbalanced, a dummy supply or a dummy demand can be added to restore balance. Once initial basic feasible solution is obtained, MODI method and Stepping Stone method [7] provides optimal basic feasible solution for the transportation problem.

In this paper, a method named Maximum Difference Method (MDM) is proposed to find the initial basic feasible solution of the transportation problem which is most of the time better than the solution obtained by Vogel's Approximation method. In the second section mathematical representation of the transportation problem is shown, alongwith the algorithm of VAM and proposed maximum difference method (MDM). In the third section numerical examples are illustrated to discuss the proposed method. Finally, a table is shown to compare the transportation costs of existing methods and the new proposed method.

## MATHEMATICAL REPRESENTATION

Let there be $m$ origins $o_i$ having $a_i (i = 1,2,\dots,m)$ units of source respectively which are to be transported to $n$ destinations $D_j$'s with $b_j (j = 1,2,\dots n)$ units of demand respectively. Let $C_{ij}$ be the cost of source one unit of commodity form origin $i$ to destination $j$. If $x_{ij}$ represents the units source from origin $i$to destination $j$ then problem is to determine the transportation schedule so as to minimize the total transportation cost satisfying supply and demand condition.

Mathematically the problem can be expressed as

Minimum $z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i$ for $i = 1,2, \dots, m$ (Supply constraints)

$\sum_{i=1}^{m} x_{ij} = b_j$ for $j = 1,2, \dots, n$ (Demand constraints)

and $x_{ij} \geq 0$ for all *i* and *j.*

A transportation problem is said to be balanced if the total supply from all sources equals the total demand in all destinations i.e., $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ otherwise it is called unbalanced.

A set of non-negative allocation $x_{ij} \geq 0$, which satisfy the row and column restriction is known as feasible solution. A feasible solution is basic if the numbers of positive allocations are $m + n - 1$. If the number of allocations are less than $m + n - 1$ then it is called degenerate feasible solution. A feasible solution is called an optimal solution if it minimizes the total transportation cost.

The existing methods to find initial basic feasible solutions are North West-Corner Method (NWCM), Least Cost Method (LCM) and Vogel's Approximation Method (VAM). It is seen that VAM provides the initial basic feasible solution which is closer to the optimal solution [8]. In this paper, we have proposed a new method named Maximum Difference Method (MDM) to find the initial basic feasible solution of the transportation problem which is either better or same as that of Vogel's Approximation method.

**Algorithm of Vogel's Approximation Method (VAM)** [9]

*Step 1:* Identify the cells having minimum and next to minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.

*Step 2:* Identify the cells having minimum and next to minimum transportation cost in each column and write the difference (penalty) along the side of the table against the corresponding column.

*Step 3*: Identify the maximum penalty. If it is along the side of the table make maximum allotment to the cell having minimum cost of transportation in that row. If it is below the table make maximum allotment to the cell having minimum cost of transportation in that column.

*Step 4*: If the penalty corresponding to two or more rows or columns are equal, select the topmost row and the extreme left corner.

*Step 5*: No further consideration is required for the row or column which is satisfied. If both the row and columns are satisfied at a time delete only one of the two and the remaining row and the column is assigned zero supply (or demand).

*Step 6*: Calculate fresh penalty for the remaining sub-matrix as in step 1 and allocate following the procedure of previous step. Continue the process until all rows and column are satisfied.

*Step 7*: Finally total minimum cost is calculated as sum of the product of cost and corresponding allocated value of supply /demand, i.e., Total Cost $= \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$.

**Algorithm of proposed Maximum Difference Method (MDM):**

*Step 1:* Identify the cells having maximum and next to maximum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.

*Step 2:* Identify the cells having maximum and next to maximum transportation cost in each column and write the difference (penalty) along the side of the table against the corresponding column.

*Step 3*: Identify the maximum penalty. If it is along the side of the table make maximum allotment to the cell having minimum cost of transportation in that row. If it is below the table make maximum allotment to the cell having minimum cost of transportation in that column.

*Step 4*: If the penalty corresponding to two or more rows or columns are equal, select the topmost row and the extreme left corner.

*Step 5*: No further consideration is required for the row or column which is satisfied. If both the row and columns are satisfied at a time delete only one of the two and the remaining row and the column is assigned zero supply (or demand).

*Step 6*: Calculate fresh penalty for the remaining sub-matrix as in step 1 and allocate following the procedure of previous step. Continue the process until all rows and column are satisfied.

*Step 7*: Finally total minimum cost is calculated as sum of the product of cost and corresponding allocated value of supply /demand, i.e., Total Cost $= \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$.

We illustrate the propose Maximum Difference Method (MDM) by the following Transportation problems

**NUMERICAL ILLUSTRATIONS**

**Example 1:** A company has four factories manufacturing the same commodity, which is required to be transported to meet the demands in four warehouses. The supplies and demands as also the cost of transportation from factory to warehouse in rupees per unit of the product are given in table as follow:

| Factories | Warehouses | | | | Supply |
|---|---|---|---|---|---|
| | W | X | Y | Z | Units |
| A | 25 | 55 | 40 | 60 | 60 |
| B | 35 | 30 | 50 | 40 | 80 |
| C | 36 | 45 | 26 | 66 | 160 |
| D | 35 | 30 | 41 | 150 | 150 |
| Demand Units | 90 | 100 | 120 | 140 | 450 |

**Table: 1.1**

**Solution applying Vogel's approximation method (VAM)**

Therefore, the initial basic feasible solution by *Vogel's Approximation Method* (VAM) is

$x_{11} = 60, x_{24} = 80, x_{33} = 100, x_{34} = 60,\ x_{41} = 30,\ x_{42} = 100, x_{43} = 20.$

The Total cost of transportation= (25x40) + (60x20) + (40x80) + (26x120) + (66x40) + (35x50) + (30x100) =15,910

| | W | X | Y | Z | Supply | Row Penalty |
|---|---|---|---|---|---|---|
| *A* | [60] 25 | 55 | 40 | 60 | 60 | (15) *(15)*× ×× |
| *B* | 35 | 30 | 50 | [80] 40 | 80 | (5) × ×× × |
| *C* | 36 | 45 | [100] 26 | [60] 66 | 160 | (10) (10) (10) (10) × |
| *D* | [30] 35 | [100] 30 | [20] 41 | 150 | 150 | (5) (5) (5) (5) (5) |
| Demand | 90 | 100 | 120 | 140 | 450 | |
| Column Penalty | (10) (10) (1) (1) | (0) 15 15 15 | (14) (14) (15) *(15)* | *(20)* (6) *(84)* × | | |

**Table: 1.2:** Solution applying Vogels approximation method (VAM)

## 3. Solution applying proposed maximum difference method (MDM)

| | *W* | *X* | *Y* | *Z* | *Supply* | *Row Penalty* |
|---|---|---|---|---|---|---|
| *A* | [40] 25 | 55 | 40 | [20] 60 | 60 | (5) (20) (20) (*35*) |
| *B* | 35 | 30 | 50 | [80] 40 | 80 | (10) (10) (10) (5) |
| *C* | 36 | 45 | [120] 26 | [40] 66 | 160 | (21) (30) (*30*) (30) |
| *D* | [50] 35 | [100] 30 | 41 | 150 | 150 | (*109*) (*109*) ×× |
| *Demand* | 90 | 100 | 120 | 140 | 450 | |
| *Column Penalty* | (1) (1) (1) (1) × | (10) × × × × | (9) (9) (10) × × | (84) (84) (6) (6) (6) | | |

**Table: 1:** Solution applying proposed maximum difference method (MDM)

The difference between the maximum and the next to maximum costs in each row and each column are computed and displayed inside the parenthesis against the respective columns and rows. The maximum difference is 109 which occurs in row D. The lowest cost of row D is 30 which is in column X. Allocate Min (150, 100) =100 in the cell (D, X), i.e.,$x_{42} = 100$ and diminish 150 by 100. Since the demand of X warehouse is exhausted completely, allocate 0 to rest of the cell of column X. Calculate the row and column difference by same technique in the reduced cost matrix and is displayed in the *Table 1.3*.

Therefore, the initial basic feasible solution by *Maximum Difference Method* (MZM) is $x_{11} = 40, x_{14} = 20, x_{24} = 80, x_{33} = 120, x_{34} = 40,\ x_{41} = 50,\ x_{42} = 100.$

The Total cost of transportation= $(25 \times 40) + (60 \times 20) + (40 \times 80) + (26 \times 120) + (66 \times 40) + (35 \times 50) + (30 \times 100) = 15{,}910$

**Example 2:** A steel company is concern with the problem of distributing important ore from four ports to six steel mills throughout the country. The transportation costs per tonne per kilometre are given below:

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ | $M_6$ | Supply (in tonnes) |
|---|---|---|---|---|---|---|---|
| $P_1$ | 9 | 12 | [5] 9 | 6 | 9 | 10 | 5 |
| $P_2$ | 7 | [4] 3 | 7 | 7 | 5 | [2] 5 | 6 |
| $P_3$ | [3] 6 | 5 | [1] 9 | 11 | 3 | 11 | 2 |
| $P_4$ | [3] 6 | 8 | 11 | [2] 2 | [4] 2 | 10 | 9 |
| Demand (in tonnes) | 4 | 4 | 6 | 2 | 4 | 2 | 22 |

**Table: 2**

Therefore, the initial basic feasible solution by *Maximum Difference Method* (MDM) is

$x_{13} = 5, x_{22} = 4, x_{26} = 2, x_{31} = 1, x_{33} = 1,\ x_{41} = 3,\ x_{44} = 2,\ x_{45} = 4.$

**The Total cost of transportation=**

$(9 \times 5) + (3 \times 4) + (5 \times 2) + (6 \times 1) + (9 \times 1) + (6 \times 3) + (2 \times 2) + (2 \times 4) = 112$

**Example 3**: Three fertilizer factories *X, Y* and *Z* are located at different places of the country Produce 11, 13 and 19 lakhs tonnes of urea respectively. They are to be distributed to four states *A, B, C* and *D* as 6,10, 12 and 15 lakhs tonnes respectively. The transportation cost per tonne in Rs. is given below:

| | *A* | *B* | *C* | *D* | *Supply (in lakhs tonnes)* |
|---|---|---|---|---|---|
| *X* | 21 | 16 | 25 | [11] 13 | 11 |
| *Y* | [6] 17 | [3] 18 | 14 | [4] 23 | 13 |
| *Z* | 32 | [7] 27 | [12] 18 | 41 | 19 |
| *Demand (in lakhs tonnes)* | 6 | 10 | 12 | 15 | 43 |

**Table: 3**

Therefore, the initial basic feasible solution by *Maximum Difference Method* (MDM) is $x_{14} = 11, x_{21} = 6, x_{22} = 3, x_{24} = 4, x_{32} = 7,\ x_{33} = 12$

The Total cost of transportation =

$(13 \times 11) + (17 \times 6) + (18 \times 3) + (23 \times 4) + (27 \times 7) + (18 \times 12) = 796$

**Example 4**: A state has five hospitals *A, B, C, D* and *E*. Their monthly requirement of medicinesetc are met by three distribution centers *X, Y* and *Z*. The data in respect of a particular item and availability at centers, and requirements at the hospital and distribution cost per unit (in Rs.) is given in the following tables:

| | *A* | *B* | *C* | *D* | *E* | *Supply (in hundreds)* |
|---|---|---|---|---|---|---|
| *X* | 6 | [15] 4 | [85] 4 | 7 | 5 | 100 |
| *Y* | [60] 5 | [65] 6 | 7 | 4 | 8 | 125 |
| *Z* | 3 | 4 | 6 | [105] 3 | [70] 4 | 175 |
| *Demand (in hundreds)* | 60 | 80 | 85 | 105 | 70 | 400 |

**Table 4**

Therefore, the initial basic feasible solution by *Maximum Difference Method* (MDM) is $x_{12} = 15, x_{13} = 85, x_{21} = 60, x_{22} = 65, x_{34} = 105,\ x_{35} = 70$

The Total cost of transportation =

$(4 \times 15) + (4 \times 85) + (5 \times 60) + (6 \times 65) + (3 \times 105) + (4 \times 70) = 1685$

**Comparison of total cost of transportation problem from various methods**

Comparison of total cost of transportation problem from various methods is shown in the following table:

| *Table No.* | *Problem size* | *MDM* | *NWCM* | *LCM* | *VAM* | *MODI* |
|---|---|---|---|---|---|---|
| *1.1* | 4x4 | 15,910 | 30,570 | 26,310 | 16,130 | 15,910 |
| *2.1* | 4x6 | 112 | 139 | 114 | 112 | 112 |
| *3.1* | 3x4 | 796 | 1095 | 922 | 796 | 796 |
| *4.1* | 3x5 | 1685 | 1950 | 1870 | 1690 | 1580 |

**Table: 5**

**CONCLUSION**

In this paper, a new algorithm for finding initial basic feasible solution of the transportation problem is developed. From *Table 5.1* it is clear that the initial basic feasible solution obtained from *Maximum Difference Method* (*MDM*) is most of the time better than that of Vogel's Approximation Method (VAM).

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## Cite this article as:

Smita Sood and Keerti Jain. The Maximum Difference Method To Find Initial Basic Feasible Solution For Transportation Problem, Asian Journal of Management Sciences 03 (07); 2015; 08-11.